Name: ______________________

AP Physics 1 Summer Work Answer Sheet

Physics and Math Primer (33)

Scientific Notation

1. ________________________
2. ________________________
3. ________________________
4. ________________________
5. ________________________
6. ________________________

Calculator Use

7. A B C D
8. A B C D
9. A B C D
10. A B C D

Unit Conversions

11. A B C D
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13. ________________________

Working with Fractions

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17. A B C D
18. A B C

Simplifying Algebraic Expressions

19. A B C
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21. A B C D

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22. A B C D
23. A B C D
24. A B C D
25. A B C D

Right Triangles

26. ________________________
27. ________________________
28. ________________________
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Inverse Trig Functions

30. A B C D
31. A B C D
32. ________________________
33. ________________________

Name: ______________________

Vector Math (21)

Pythagorean Theorem

1. ________________________
2. ________________________
3. A B C D E
4. ________________________
5. ________________________
6. ________________________

Trig Functions & Right Triangles

7. ________________________
8. ________________________
9. ________________________
10. ________________________

Vector Addition

11. ________________________
12. A B C D
13. A B C D E
14. ________________________
15. ________________________

Components of Vectors

16. ________________________
17. ________________________
18. ________________________
19. ________________________
20. ________________________
21. ________________________

Proportional Reasoning (13)

Proportional Reasoning

1. ________________________
2. ________________________
3. ________________________
4. ________________________
5. A B C D E
6. A B C D E

Parent Graphs

7. A B C D E F
8. A B C D E F
9. A B C D E F G H
10. A B C D E F G H
11. A B C D E F G H
12. A B C D E F G H
13. 1. _______ 2. ________ 3.__________ 4. __________

33 questions

Physics and Math Primer

A Welcome to the Physics Primer!

Students who successfully complete this unit will be able to:

- Describe the purpose of the Physics Primer.

What is in the Physics Primer?

The Physics Primer is not a comprehensive online mathematics textbook. It is a focused set of mathematics topics that can give new physics students trouble. Remember, mathematics is only *part* of the physics problem-solving process, but it is a part that often trips up students. Launch the short video called "Thinking Like a Physicist" to learn more about how the math skills covered in this primer relate to success in a physics course.

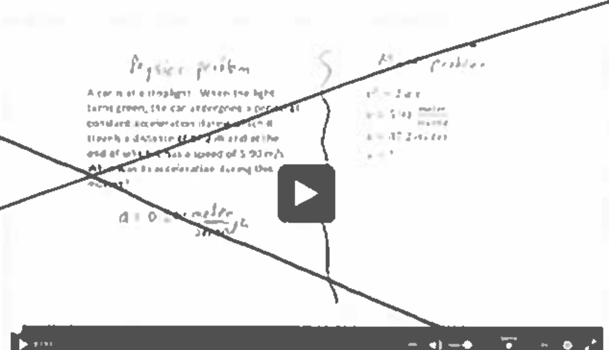
The Physics Primers include:

- Learning goals and the necessary prerequisite math skills (if applicable).
- Succinct explanations of the topic that are pertinent to a physics course.
- The proper amount of math needed to apply to relevant physics topics.
- Hints and feedback that are based on common mistakes that students make.
- Short video presentations that supplement some of the topics.

How do you use the Physics Primer?

Your physics instructor will assign some, or even all, topics based on the needs of your specific course. These may be assigned at the beginning of your course or throughout your course as appropriate. When beginning one of the topics, carefully read through the introductory material before attempting the problems that follow. If you encounter trouble along the way, don't hesitate to go back and re-read the material as well as take advantage of the available hints, which are there to help guide you in the right direction. In addition, depending on the topic, there may be an accompanying video that goes into further detail with a worked example, so be sure to utilize them as well!

Click here to watch a video on the relationship between physics and mathematics.



Part A - Applying what you just learned

What is the Physics Primer?

You did not open hints for this part.

ANSWER:

- ~~A tutorial of the most important physics concepts covered in introductory physics courses.~~
- ~~A list of the most important formulas used in introductory physics courses.~~
- ~~A set of mathematics topics that are relevant to introductory physics courses.~~
- ~~An detailed explanation of all of the mathematics you will use in your introcuctory physics course.~~

Scientific Notation

Students who successfully complete this primer will be able to:

- Convert both very large and very small numbers into scientific notation.
- Convert numbers from scientific notation into regular notation.
- Multiply and divide numbers written in scientific notation.

Before working on this primer, you may need to review:

- Writing values as powers of 10

For additional practice, you may want to review Conversion to Scientific Notation.

Why write numbers in scientific notation

Understanding physics will help you describe in great detail how the world works. However, the characteristics of the world span an enormous range of numbers. The mass of an electron is 0.000000000000000000000000000000911 $\mathbf{kg}$ (incredibly tiny!). The mass of the Sun is 1,989,000,000,000,000,000,000,000,000,000 $\mathbf{kg}$ (incredibly massive!). If you had to write, interpret, or use these numbers as written, you could very easily end up making a minor math error. Scientific notation was invented to easily express numbers that are too large or too small to be conveniently written in decimal form. The advantages of scientific notation are it 1) is compact, 2) helps articulate significant figures, and 3) works with numbers of any size.

The traditional format for numbers written in scientific notation is $m \times 10^n$ is where m is a number between 1 and 10 and n is an integer (either positive or negative). For the examples above, the mass of an electron can be written as 9.11×10^{-31} $\mathbf{kg}$ and the mass of the Sun can be written as 1.989×10^{30} $\mathbf{kg}$.

<u>Converting decimals to scientific notation</u>

A day has 86,400 second (s). Notice that the decimal point is to the far right of the number. The first part of scientific notation is a number between 1 and 10. Move the decimal point to the left to obtain 8.64, which equals m in our traditional notation. The value for n is given by the number of places the decimal point was moved from its location in 86,400 to 8.64. Thus, 86,400 s = 8.64×10^4 s. Because the decimal was moved four places to the left (and the value is larger than 1), the exponent is a positive integer. If you had a very precise clock, you could write this as 8.640×10^4 s, or 8.6400×10^4 s. The number of digits you keep in the first part of the notation describes the precision of your value and determines the number of significant digits.

As another example, the fastest glacier in the world has an average speed of 0.0012 mile/hour. The first part of scientific notation is a number between 1 and 10. Move the decimal point to the right to obtain 1.2, which equals m in our traditional notation. The value for n is given by the number of places the decimal point was moved from its location in 0.0012 to 1.2. Thus, 0.0012 mile/hour = 1.2×10^{-3} mile/hour. Because the decimal was moved three places to the right, and the value is smaller than 1, the exponent is a negative integer.

Part A - Write this large number in scientific notation.

Determine the values of m and n when the following mass of the Earth is written in scientific notation: 5,970,000,000,000,000,000,000,000 kg.

Enter m and n, separated by commas.

ANSWER:

1. m, n =

Part B - Write this large number in scientific notation.

Determine the values of m and n when the following average distance from the Sun to the Earth is written in scientific notation: 150,000,000,000 m.

Enter m and n, separated by commas.

Hint 1. Moving the decimal point

Move the decimal point to the left so you end up with a number between 1 and 10. That's the value for m .

Hint 2. Finding n

Count the number of place values you moved the decimal point.

Hint 3. Sign of the exponent

For a value greater than 1, the exponent is positive.

ANSWER:

2. m, n =

Part C - Write this small number in scientific notation.

Determine the values of m and n when the following charge of a proton is written in scientific notation: 0.000000000000000000160 C.

Enter m and n, separated by commas.

Hint 1. Moving the decimal point

Move the decimal point to the right so you end up with a number between 1 and 10. That's the value for m .

Hint 2. Finding n

Count the number of place values you moved the decimal point.

Hint 3. Sign of the exponent

For a value between 0 and 1, the exponent is negative.

ANSWER:

3. m, n =

Part D - Write this small number in scientific notation.

Determine the values of m and n when the following average magnetic field strength of the Earth is written in scientific notation: 0.0000451 T.

Enter m and n, separated by commas.

Hint 1. Moving the decimal point

Move the decimal point to the right so you end up with a number between 1 and 10. That's the value for m .

Hint 2. Finding n

Count the number of place values you moved the decimal point.

Hint 3. Sign of the exponent

For a value between 0 and 1, the exponent is negative.

ANSWER:

4. m, n =

Rule for multiplying numbers in scientific notation

When multiplying two numbers of the form $m \times 10^n$, the product is: $(m_1 \times 10^{n_1})(m_2 \times 10^{n_2}) = m_1 m_2 \times 10^{n_1+n_2}$.

Part E - Practice multiplying numbers in scientific notation

The mass of a high speed train is 4.5×10^5 kg, and it is traveling forward at a velocity of 8.3×10^1 $\mathrm{m/s}$. Given that momentum equals mass times velocity, determine the values of m and n when the momentum of the train (in $\mathrm{kg} \cdot \mathrm{m/s}$) is written in scientific notation.

Enter m and n, separated by commas.

Hint 1. Rule for multiplying orders of magnitude

Multiply the orders of magnitude by adding the exponents.

Hint 2. Rule for finding m

Don't forget that m must be between 1 and 10.

ANSWER:

5. m, n =

Rule for dividing numbers in scientific notation

When dividing two numbers of the form $m \times 10^n$, the quotient is: $(m_1 \times 10^{n_1}) \div (m_2 \times 10^{n_2}) = \frac{m_1}{m_2} \times 10^{n_1 - n_2}$.

Part F - Practice dividing numbers in scientific notation

Given that average speed is distance traveled divided by time, determine the values of m and n when the time it takes a beam of light to get from the Sun to the Earth (in s) is written in scientific notation. Note: the speed of light is approximately $3.0 \times 10^8 \ \mathrm{m/s}$.

Enter m and n, separated by commas.

Hint 1. Sun to Earth distance

Recall the average distance between the Sun and the Earth from part B.

Hint 2. Rule for finding n

Divide the orders of magnitude by subtracting the exponents.

ANSWER:

6. $m, n =$

Calculator Use

Students who successfully complete this primer will be able to:

- Recognize the importance in learning the rules for the particular calculator being used
- Learn specific features of calculators that will be commonly used
- Employ a strategy for using a calculator that reduces the chance of making errors
- Use a calculator to evaluate numerical expressions commonly encountered in physics

Note: it is strongly recommended that you work through this unit **using the specific calculator** that you will use in class, on homework assignments, and for tests!

For additional practice, you may want to review Evaluating Powers of 10, or Solving Radical Equations.

Overview of methods for correctly using a calculator

1. Different calculators operate in different ways. The order in which numbers, operations, and functions are entered, the markings of the keys that perform functions, the way the information is displayed, and the way the calculator is set to interpret information entered (e.g., whether angles are in degrees or radians), can all vary widely from calculator to calculator. Therefore, it is highly recommended that you make a habit of using the specific calculator with which you are familiar for all of the work in your physics course.

2. The order in which information is entered into the calculator and operations are performed **matters**! This is called "precedence of operation," and you must familiarize yourself with it. In general, you **cannot** simply enter an expression into a calculator in the order you read it and obtain the correct result! A very helpful strategy for avoiding

precedence of operation errors when evaluating longer expressions with a calculator is to perform intermediate calculations one step at a time:

- Select smaller portions of the whole expression.
- Use the calculator to perform these intermediate evaluations.
- Rewrite the expression using the intermediate results.
- Proceed until the entire expression has been evaluated.

3. Below are calculator keys and operations which you must be able to use correctly for a physics class:

- **Clear key** (you may need to use different keys to clear the display and clear values from the memory of the calculator)
- **Number keys** $(0-9)$, including the **decimal point** $(.)$
- **Operation keys** $(+, -, \times, \div, =)$
- **Change sign key** (note that on some calculators the change sign key looks very similar to the subtraction key; these two keys do NOT do the same thing so you must be careful to distinguish between them)
- **Square root** $(\sqrt{\ })$, **square** (x^2), **inverse** $(1/x$ or $x^{-1})$, and **exponent** $(x^y$ or $\wedge)$ operations. You may need to use a **shift** (or **2nd**) **key** on the calculator for some of these operations.
- Changing the angle units between **degrees** (deg) and **radians** (rad) (some calculators display the angle units currently set, others don't, although you should be able to change these units in your calculator's settings)
- Trigonometric functions: **sine** $(\sin)$, **cosine** $(\cos)$, and **tangent** $(\tan)$, as well as inverse trigonometric functions: **arcsine** $(\arcsin$, or $\sin^{-1})$, **arccosine** $(\arccos$, or $\cos^{-1})$, and **arctangent** $(\arctan$, or $\tan^{-1})$. You may need to use a shift (or 2nd) key for some of these operations. (Note: make sure you must use the correct setting of angle units for both trigonometric functions and inverse trigonometric functions)
- Scientific notation: you need to know the key that you use to enter numbers in scientific notation (e.g. **E**, **EE**, or **EXP**), and the way in which your calculator displays numbers that are expressed in scientific notation.
- Other keys that may also be useful: **parentheses** (which can be used to group terms and reduce precedence of operation errors), **constants** (such as π), **memory** to store intermediate results, **logarithm** $(\log)$ and **natural logarithm** $(\ln)$, **statistics** (**STAT**), and more.

Learn your calculator (example 1)

Let's evaluate the quantity $\bar{v} = \dfrac{15 \text{ meter} + 5 \text{ meter}}{2 \text{ second} + 3 \text{ second}}$. You can also do this evaluation in your head. That's an important strategy for verifying that you are using your calculator correctly! Let's apply the one-step-at-a-time strategy. First, perform the addition in the numerator using the calculator:

$$\bar{v} = \frac{20 \text{ meter}}{2 \text{ second} + 3 \text{ second}}.$$

Then, use the calculator to perform the addition in the denominator:

$$\bar{v} = \frac{20 \text{ meter}}{5 \text{ second}}.$$

Finally, use the calculator to perform the division to obtain the correct final numerical answer:

$$\bar{v} = 4 \text{ meter/second}.$$

Note that if you enter the values and operations into your calculator in exactly the same order that you read them in the expression, $15 + 5 \div 2 + 3$, your calculator will display the wrong answer for the expression. This would be a precedence of operation error! An alternative approach in a case like this is to place parentheses around the terms in the numerator and denominator separately in your calculator: $(15 + 5) \div (2 + 3)$. When entered this way, your calculator, which follows the order of operations, gives the correct answer. Now try a problem on your own.

Part A - Average acceleration

Use your calculator to evaluate $\bar{a} = \dfrac{-3.7 \text{ meter/second} - 13.9 \text{ meter/second}}{21.4 \text{ second} - 7.2 \text{ second}}$.

You did not open hints for this part.

ANSWER:

7.

A $11.5 \text{ meter/second}^2$

B $-1.24 \text{ meter/second}^2$

C $-0.718 \text{ meter/second}^2$

D $0.718 \text{ meter/second}^2$

Learn your calculator (example 2)

Imagine we wish to evaluate the quantity $\frac{1}{2}(1.67 \times 10^{-27} \text{ kilogram})(3.00 \times 10^5 \text{ meter/second})^2$. Start by verifying the operations of your calculator to work with scientific notation with an expression for which you know the answer: $2 \times (5 \times 10^1)$, and then verify the squaring operation of your calculator with 3^2. Finally, once you've completed this, use your calculator to evaluate the original expression, which should come out to $7.52 \times 10^{-17} \text{ kilogram} \cdot \text{meter}^2/\text{second}^2$.

Learn your calculator (example 3)

Imagine we wish to evaluate the quantity $\sqrt{2 \times 4.71 \frac{\text{meter}}{\text{second}^2} \times 12.9 \text{ meter}}$. Start by verifying the square root operation with $\sqrt{100}$. On some calculators, you enter the square root operator first and then the number (sometimes followed by a closed parenthesis), while on others you enter the number first and *then* the square root operator. The same may be true for other operations as well (e.g. trigonometric and inverse trigonometric functions, inverse functions, and so on). Make sure you learn how **your** calculator works with these important functions and operators! Remember, also, units are central to physics– **propagate them into your answer**. Finally, once you've completed this, use your calculator to evaluate the original expression, which should come out to $11.0 \text{ meter/second}$. Try another problem now on your own.

Part B - Astronomy calculation

In analyzing distances by applying the physics of gravitational forces, an astronomer has obtained the expression

$$R = \sqrt{\frac{1}{\left(\frac{1}{149 \times 10^9 \text{ meter}}\right)^2 - \left(\frac{1}{298 \times 10^9 \text{ meter}}\right)^2}}$$

Use your calculator to evaluate R.

Hint 1. Verify proper use of the calculator

Make sure you verify how your calculator performs the necessary operations using it to calculate expressions for which you know the answer.

Hint 2. One-step-at-a-time

Use the one-step-at-a-time strategy

ANSWER:

8.

A 1.49×10^{11} meter

B 1.72×10^{11} meter

C 3.87×10^{-12} meter

D 2.58×10^{11} meter

Learn your calculator (example 4)

Imagine we wish to evaluate the quantity $\frac{(17.2\ \mathrm{meter/second})^2}{2 \times 9.80\ \mathrm{meter/second}^2 \times \sin 40^\circ}$. Start by verifying the **sine** function operation **and** that the angle units of the calculator are set to degrees by confirming $\sin 90^\circ$ comes out to 1. Common ways to set angle units are with a **MODE** or **DGR** key. On some calculators you enter the trigonometric operator first and then the angle (sometimes followed by a closed parenthesis), while on others you enter the angle first and *then* the trigonometric operator. Finally, once you've completed this, use your calculator to evaluate the original expression, which should come out to $23.5\ \mathrm{meter}$.

Learn your calculator (example 5)

Imagine we wish to evaluate the quantity $\arctan\left(\frac{7.00\ \mathrm{meter}}{4.00\ \mathrm{meter}}\right)$. Start by verifying the **arctangent** function and that the angle units of the calculator are set to degrees just as above by confirming $\arctan(1)$ comes out to 45°. Note that this function will likely be written as $\tan^{-1}$ on your calculator. Finally, once you've completed this, use your calculator to evaluate the original expression, which should come out to 60.3°, then try the last two problems.

Part C - Projectile motion

A student solving a physics problem for the range of a projectile has obtained the expression

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

where $v_0 = 37.2\ \mathrm{meter/second}$, $\theta = 14.1^\circ$, and $g = 9.80\ \mathrm{meter/second}^2$. Use your calculator to evaluate R.

Hint 1. Inserting values into the equation

Write down the expression with the given values and units replacing the variables.

Hint 2. Verify proper use of the calculator

Make sure you verify how your calculator performs the necessary operations using it to calculate expressions for which you know the answer.

Hint 3. One-step-at-a-time

Use the one-step-at-a-time strategy, where one step will be to multiply θ by 2 **before** you evaluate the sine function.

ANSWER:

9.

A 66.7 meter

B 10.5 meter

C 69.5 meter

D 31.5 meter

Part D - Capstone problem

A group of physics students hypothesize that for an experiment they are performing, the speed of an object sliding down an inclined plane will be given by the expression

$$v = \sqrt{2gd(\sin(\theta) - \mu_k \cos(\theta))}.$$

For their experiment, $d = 0.725$ meter, $\theta = 45.0°$, $\mu_k = 0.120$, and $g = 9.80$ meter/second^2. Use your calculator to obtain the value that their hypothesis predicts for v.

Hint 1. Inserting values into the equation

Write down the expression with the given values and units replacing the variables.

Hint 2. Verify proper use of the calculator

Make sure you verify how your calculator performs the necessary operations using it calculate expressions for which you know the answer.

Hint 3. One-step-at-a-time

Use the one-step-at-a-time strategy.

ANSWER:

10.

A 2.43 meter/second

B 2.97 meter/second

C 3.15 meter/second

D 6.25 meter/second

Unit Conversions

Students who successfully complete this primer will be able to:

- Identify factors associated with physically important metric prefixes.
- Convert units between quantities possessing simple/compound units, multiple conversion factors, and metric prefixes.

Before working on this primer you may need to review:

- Scientific notation

For additional practice, you may want to review Conversion Factors.

Overview of unit conversion steps

1. Identify the equality relationship between the quantities that are being converted (e.g. $1\ \text{mile} = 5280\ \text{foot}$). Metric prefixes (see the table below) are just a shorthand for an equality relationship between units, and are based on powers of 10 (e.g. $1\ \text{millimeter} = 10^{-3}\ \text{meter}$)
2. Use the equality relationship to obtain a **conversion factor** that is a ratio of the units. Since the numerator and denominator of a conversion factor are equal, it will always have a value of exactly 1. Therefore, when you multiply a quantity by a conversion factor, it will not change its inherent value, rather just the associated units. The conversion factor can have ***either*** of the units in the numerator, with the other in the denominator. The placement of the units depends on the quantity being converted. (e.g. $\frac{5280\ \text{foot}}{1\ \text{mile}}$ or $\frac{1\ \text{mile}}{5280\ \text{foot}}$)
3. Multiply the quantity to be converted by the appropriate factor obtained in the step above. If the same unit appears in the numerator ***and*** denominator of a fraction, it cancels. Therefore, write your conversion factor such that the unit being converted cancels out. Be sure to write these conversion expressions with ***both*** the numerical quantities ***and*** the units so that they can be correctly evaluated (e.g. $14,411\ \text{foot} \times \frac{1\ \text{mile}}{5280\ \text{foot}} = 2.7294\ \text{mile}$)
4. To convert quantities with compound units, apply multiple factors using the steps above. To check that the units being converted have all cancelled out and only the desired units remain, you may find it helpful to cross off the cancelled units along the way (e.g.
$1.20\ \frac{\cancel{\text{mile}}}{\cancel{\text{minute}}} \times \frac{5280\ \text{foot}}{1\ \cancel{\text{mile}}} \times \frac{1\ \cancel{\text{minute}}}{60\ \text{second}} = 106\ \frac{\text{foot}}{\text{second}}$ or, for units ***raised to a power***
$7.29 \times 10^{6}\ \cancel{\text{millimeter}^2} \times \frac{10^{-3}\ \text{meter}}{1\ \cancel{\text{millimeter}}} \times \frac{10^{-3}\ \text{meter}}{1\ \cancel{\text{millimeter}}} = 7.29\ \text{meter}^2$)

Below is a table of important metrix prefixes:

Prefix (abbreviation)	**Power of 10**	**Equality relationship**	**Example**
nano (n)	10^{-9}	$1\ \text{nano}\mathit{unit} = 10^{-9}\ \mathit{unit}$	$632\ \text{nanometer} = 632.8 \times 10^{-9}\ \text{meter}$
micro (μ)	10^{-6}	$1\ \text{micro}\mathit{unit} = 10^{-6}\ \mathit{unit}$	$5.1\ \text{microsecond} = 5.1 \times 10^{-6}\ \text{second}$
milli (m)	10^{-3}	$1\ \text{milli}\mathit{unit} = 10^{-3}\ \mathit{unit}$	$50\ \text{milliKelvin} = 50 \times 10^{-3}\ \text{Kelvin}$
centi (c)	10^{-2}	$1\ \text{centi}\mathit{unit} = 10^{-2}\ \mathit{unit}$	$2.54\ \text{centimeter} = 2.54 \times 10^{-2}\ \text{meter}$
kilo (k)	10^{3}	$1\ \text{kilo}\mathit{unit} = 10^{3}\ \mathit{unit}$	$75\ \text{kilogram} = 75 \times 10^{3}\ \text{gram}$
mega (M)	10^{6}	$1\ \text{mega}\mathit{unit} = 10^{6}\ \mathit{unit}$	$0.778\ \text{megaparsec} = 0.778 \times 10^{6}\ \text{parsec}$
giga (G)	10^{9}	$1\ \text{giga}\mathit{unit} = 10^{9}\ \mathit{unit}$	$10\ \text{gigabyte} = 10 \times 10^{9}\ \text{byte}$

Part A - Obtaining conversion factors from unit equality relationships

In an experiment you are performing, your lab partner has measured the distance a cart has traveled: $28.4\ \text{inch}$. You need the distance in units of centimeter and you know the unit equality $1\ \text{inch} = 2.54\ \text{centimeter}$. By which conversion factor will you multiply $28.4\ \text{inch}$ in order to perform the unit conversion?

You did not open hints for this part.

ANSWER:

11.

A $\dfrac{1\ \text{centimeter}}{2.54\ \text{inch}}$

B $\dfrac{2.54\ \text{centimeter}}{1\ \text{inch}}$

C $\dfrac{1\ \text{inch}}{2.54\ \text{centimeter}}$

D $\dfrac{2.54\ \text{inch}}{1\ \text{centimeter}}$

Part B - Converting compound units

You would like to know whether silicon will float in mercury and you know that can determine this based on their densities. Unfortunately, you have the density of mercury in units of $\frac{\text{kilogram}}{\text{meter}^3}$ and the density of silicon in other units: $2.33\ \frac{\text{gram}}{\text{centimeter}^3}$. You decide to convert the density of silicon into units of $\frac{\text{kilogram}}{\text{meter}^3}$ to perform the comparison. By which combination of conversion factors will you multiply $2.33\ \frac{\text{gram}}{\text{centimeter}^3}$ to perform the unit conversion?

You did not open hints for this part.

ANSWER:

12.

A $\dfrac{1\ \text{kilogram}}{10^3\ \text{gram}} \times \dfrac{1\ \text{centimeter}}{10^{-2}\ \text{meter}} \times \dfrac{1\ \text{centimeter}}{10^{-2}\ \text{meter}} \times \dfrac{1\ \text{centimeter}}{10^{-2}\ \text{meter}}$

B $\dfrac{10^3\ \text{gram}}{1\ \text{kilogram}} \times \dfrac{1\ \text{centimeter}}{10^{-2}\ \text{meter}}$

C $\dfrac{10^3\ \text{gram}}{1\ \text{kilogram}} \times \dfrac{10^{-2}\ \text{meter}}{1\ \text{centimeter}} \times \dfrac{10^{-2}\ \text{meter}}{1\ \text{centimeter}} \times \dfrac{10^{-2}\ \text{meter}}{1\ \text{centimeter}}$

D $\dfrac{1\ \text{kilogram}}{10^3\ \text{gram}} \times \dfrac{1\ \text{centimeter}}{10^{-2}\ \text{meter}}$

Part C - Capstone unit conversion

You have negotiated with the Omicronians for a base on the planet Omicron Persei 7. The architects working with you to plan the base need to know the acceleration of a freely falling object at the surface of the planet in order to adequately design the structures. The Omicronians have told you that the value is $g_{OP7} = 7.29\ \frac{\text{flurg}}{\text{grom}^2}$, but your architects use the units $\frac{\text{meter}}{\text{second}^2}$, and from your previous experience you know that both the Omicronians and your architects are *terrible* at unit conversion. Thus, it's up to you to do the unit conversion. Fortunately, you know the unit equality relationships: $5.24\ \text{flurg} = 1\ \text{meter}$ and $1\ \text{grom} = 0.493\ \text{second}$. What is the value of g_{OP7} in the units your architects will use, in $\frac{\text{meter}}{\text{second}^2}$?

ANSWER:

13.

g_{OP7} = $\frac{\text{meter}}{\text{second}^2}$

Working with Fractions

Students who successfully complete this primer will be able to:

- Add, subtract, multiply and divide physical quantities that are expressed as fractions.
- Evaluate the numerical value of physical quantities expressed as fractions.

For additional practice, you may want to review Adding Fractions.

Problem solving with variables

First, it is worth pointing out the concept of solving problems using variables. Successful physics students develop the habit of performing algebraic operations when solving a problem using variables (i.e. symbols) as much as possible, rather than substituting in numerical values given in a problem and solving . There are a several very important reasons for this. Performing the algebraic operations to solve a problem using variables

- is easier, faster, and less prone to errors. Students who haven't developed the habit of using variables sometimes don't believe this because they are less familiar with the approach, but with just a little effort and practice using variables rather than numerical values, they will find it true.
- can provide some very easy to use and important checks that a problem has been solved correctly.
- gives results that are much more informative than results found by early substitution of the numerical values.

Overview of working with fractions

Below are some important properties to keep in mind related to fractions:

- Fraction notation: $\frac{\text{Numerator}}{\text{Denominator}}$
- Units in fractions: units in fractions are treated similarly to variables or numerical values with respect to multiplication, division and cancellation, and it is very important to correctly carry out the operations with the units as well as the variables or numerical values.
- Multiplying two fractions:
  - The numerator of the resulting fraction is the product of the numerators of the two fractions, while the denominator of the resulting fraction is the product of the denominators of the two fractions. (e.g. $\frac{a}{g} \cdot \frac{m_1}{m_2} = \frac{a \cdot m_1}{g \cdot m_2}$)
  - Note that a quantity that is not written in fraction form can be easily written in fraction form by writing the quantity as the numerator of a fraction whose denominator is 1.
  - *Example 1*: $m \cdot \frac{v^2}{r} = \frac{m}{1} \cdot \frac{v^2}{r} = \frac{mv^2}{r}$
  - *Example 2*: $2 \text{ kilogram} \cdot \frac{2 \text{ meter}}{7 \text{ second}} = \frac{4 \text{ kilogram} \cdot \text{meter}}{7 \text{ second}}$
- Dividing one fraction by another: Invert the fraction in the denominator and then multiply it by the fraction in the numerator.
  - *Example 1*: $\frac{\frac{11 \text{ meter}}{15 \text{ second}}}{\frac{2 \text{ second}}{5}} = \frac{11 \text{ meter}}{15 \text{ second}} \cdot \frac{5}{2 \text{ second}} = \frac{55 \text{ meter}}{30 \text{ second}^2} = \frac{11 \text{ meter}}{6 \text{ second}^2}$
  - *Example 2*: $\frac{\frac{x}{t_1}}{t_2} = \frac{\frac{x}{t_1}}{\frac{t_2}{1}} = \frac{x}{t_1} \cdot \frac{1}{t_2} = \frac{x}{t_1 t_2}$
- Adding or subtracting two fractions:
  - Multiply each fraction by a factor that equals 1 so that they have the same denominator (i.e., find a common denominator). A simple and general way to do this is to multiply the first fraction by a fraction

that has the denominator of the second fraction in both its numerator and denominator, and multiply the second fraction by a fraction that has the denominator of the first fraction in both its numerator and denominator.

- The numerator of the resulting fraction is the sum or difference of the numerators of the two fractions; the denominator of the resulting fraction is just the common denominator.
- *Example*: $\frac{2d}{3} - \frac{2d}{5} = \left(\frac{5}{5} \cdot \frac{2d}{3}\right) - \left(\frac{3}{3} \cdot \frac{2d}{5}\right) = \frac{10d}{15} - \frac{6d}{15} = \frac{10d-6d}{15} = \frac{4d}{15}$

Part A - Finding acceleration (I)

A student solving for the acceleration of an object has applied appropriate physics principles and obtained the expression $a = a_1 + \frac{F}{m}$ where $a_1 = 3.00\ \mathrm{meter/second}^2$, $F = 12.0\ \mathrm{kilogram \cdot meter/second}^2$ and $m = 7.00\ \mathrm{kilogram}$. First, which of the following is the correct step for obtaining a common denominator for the two fractions in the expression in solving for a?

You did not open hints for this part.

ANSWER:

14.

$a =$

A $\left(\frac{m}{m} \cdot \frac{a_1}{1}\right) + \left(\frac{1}{1} \cdot \frac{F}{m}\right)$

B $\left(\frac{m}{m} \cdot \frac{a_1}{1}\right) + \left(\frac{m}{m} \cdot \frac{F}{m}\right)$

C $\left(\frac{1}{m} \cdot \frac{a_1}{1}\right) + \left(\frac{1}{m} \cdot \frac{F}{m}\right)$

D $\left(\frac{m}{m} \cdot \frac{a_1}{1}\right) + \left(\frac{F}{F} \cdot \frac{F}{m}\right)$

Part B - Finding acceleration (II)

Next, based on the correct answer from Part A, which of the following is the correct symbolic expression for a?

Hint 1. Multiplying fractions

To multiply two fractions, the result is a fraction whose numerator is the product of the two numerators, and whose denominator is the product of the two denominators.

Hint 2. Combining fractions (denominator)

If two fractions are being added and they have a common denominator, the denominator of the result will be the common denominator.

Hint 3. Combining fractions (numerator)

If two fractions are being added and they have common denominator, the numerator of the result will be the sum of the numerators.

ANSWER:

15.

$a =$

A $\frac{ma_1+F}{2m}$

B $\frac{m(a_1+F)}{2m}$

C $\frac{a_1+F}{m}$

D $\frac{ma_1+F}{m}$

Part C - Finding acceleration (III)

Finally, what is the numerical vaue of a?

Hint 1. Units in fractions

Units in fractions are treated similarly to variables or numerical values with respect to multiplication, division and cancellation.

ANSWER:

16.

$a =$

A $22.7\ \mathrm{meter/second^2}$

B $4.71\ \mathrm{kilogram \cdot meter/second^2}$

C $22.7\ \mathrm{kilogram \cdot meter/second^2}$

D $4.71\ \mathrm{meter/second^2}$

E $4.71\ \mathrm{kilogram}$

Part D - Capstone problem

A student solving a physics problem for the velocity of an object has applied appropriate physics principles and obtained the expression $v = \frac{d}{t} - v_0$, where $d = 35.0\ \mathrm{meter}$, $t = 9.00\ \mathrm{second}$, and $v_0 = 3.00\ \mathrm{meter/second}$. What is the velocity v?

ANSWER:

17.

A $\frac{8.00\ \mathrm{meter}}{9.00\ \mathrm{second}}$

B $\frac{32.0\ \mathrm{meter}}{9.00\ \mathrm{second}}$

C $\frac{32.0\ \mathrm{meter}}{9.00\ \mathrm{second}}$

D $\frac{312\ \mathrm{meter}}{9.00\ \mathrm{second}}$

Simplifying Algebraic Expressions

Students who successfully complete this primer will be able to:

- Understand that expert physics problem-solvers associate quantities with variables and solve for unknowns in terms of these variables.
- Simplify algebraic relationships for expressions that are commonly encountered in physics.

For additional practice, you may want to review Rearrangement of Algebraic Expressions.

It is widely believed that it is easier to work physics problems by substituting numerical values in at the very beginning of the problem. However, expert physics problem-solvers will typically solve physics problems using variables, and substitute known values only as a *final* step, because this approach is generally faster, easier, reduces mistakes, and results in a better and more general understanding of physics. This approach may take a little practice and effort initially, but it has important benefits. Throughout this primer are several techniques that are useful for simplifying physical relationships expressed in terms of variables.

<u>**Technique 1**</u> : Substitute in known values of 0 that will simplify expressions (this is the exception to the rule that known numerical values should only be substituted in at the final step).

Example: Analysis of the physics of a system indicates that an appropriate expression describing it is given by: $x = x_0 + v_0 t + \frac{1}{2}at^2$. The problem requires solving for a, and the known values for the system are $x = 3.50 \text{ meter}$, $x_0 = 1.50 \text{ meter}$, $v_0 = 0 \text{ meter/second}$, and $t = 0.639 \text{ second}$. In this case, substituting in the known value of 0 would be the next step in the analysis, resulting in: $x = x_0 + \frac{1}{2}at^2$.

Part A

Imagine you derive the following expression by analyzing the physics of a particular system: $v^2 = v_0^2 + 2ax$. The problem requires solving for x, and the known values for the system are $a = 2.55 \text{ meter/second}^2$, $v_0 = 21.8 \text{ meter/second}$, and $v = 0 \text{ meter/second}$. Perform the next step in the analysis.

You did not open hints for this part.

ANSWER:

18.

A $0 = v_0^2 + 2ax$

B $v^2 = v_0^2 + 2ax$ (no simplification should be performed on the expression in this situation)

C $v^2 = (21.8 \text{ meter/second})^2 + 2(2.55 \text{ meter/second}^2)x$

<u>**Technique 2**</u> : Pull out common factors.

Example: Assume that the following expression is derived by analyzing the physics of a particular system: $F = ma - mg$. By pulling out the common factor, the expression for F can be simplified by writing: $F = m(a - g)$.

Part B

Imagine you derive the following expression by analyzing the physics of a particular system: $a = g\sin\theta - \mu_k g\cos\theta$, where $g = 9.80 \text{ meter/second}^2$. Simplify the expression for a by pulling out the common factor.

Hint 1. Common factors

Pull out common factors.

ANSWER:

19.

A $a = g(\sin\theta - \mu_k \cos\theta)$

B $a = g\sin\theta - \mu_k g\cos\theta$ (no simplification should be performed on the expression in this situation)

C $a = (9.80\ \mathrm{meter/second^2})\sin\theta - \mu_k(9.80\ \mathrm{meter/second^2})\cos\theta$

<u>Technique 3</u> : Reduce compound fractions (note: compound fractions are fractions that have a fraction in the numerator and/or the denominator).

Example: Assume that the following expression is derived by analyzing the physics of a particular system: $t = \frac{v}{\frac{F}{m}}$. By reducing the compound fraction, the expression for t can be simplified as: $t = \frac{mv}{F}$.

<u>Technique 4</u> : Consolidate and/or divide out factors (sometimes referred to as "cancelling") if they occur multiple times in a single expression.

Example: Assume that the following equation is derived by analyzing the physics of a particular system: $d = \sqrt{2ax}\sqrt{\frac{2x}{a}}$. By consolidating and canceling variables, the expression for d can be simplified as: $d = 2x$.

Part C

Imagine you derive the following expression by analyzing the physics of a particular system: $M = \frac{(\frac{mv^2}{r})}{(\frac{mG}{r^2})}$. Simplify the expression for M using the techniques mentioned above.

Hint 1. Dividing by a fraction

Dividing by a fraction is performed by inverting it and then multiplying by it.

ANSWER:

20.

A $M = \frac{v^2 r}{G}$

B $M = \frac{(\frac{mv^2}{r})}{(\frac{mG}{r^2})}$ (no simplification should be performed on the expression in this situation)

C $M = \frac{m^2 v^2 G}{r^3}$

Part D

A student solving a physics problem to find the unknown has applied physics principles and obtained the expression: $\mu_k mg\cos\theta = mg\sin\theta - ma$, where $g = 9.80\ \mathrm{meter/second^2}$, $a = 3.60\ \mathrm{meter/second^2}$, $\theta = 27.0°$, and m is not given. Which of the following represents a simplified expression for μ_k?

ANSWER:

21.

A $\tan\theta - \frac{a}{g}$

B $\frac{g\sin\theta - a}{g\cos\theta}$

C The single equation has two unknowns and cannot be solved with the information given.

D To avoid making mistakes, the expression should not be simplified until the numerical values are substituted.

Interpreting Graphs - Lines

Students who successfully complete this primer will be able to:

- Identify what the different parts of the slope-intercept form of a line equation correspond to on a graph.
- Determine what the slope of a position vs. time graph tells you about velocity.
- Determine what the slope of a velocity vs. time graph tells you about acceleration.

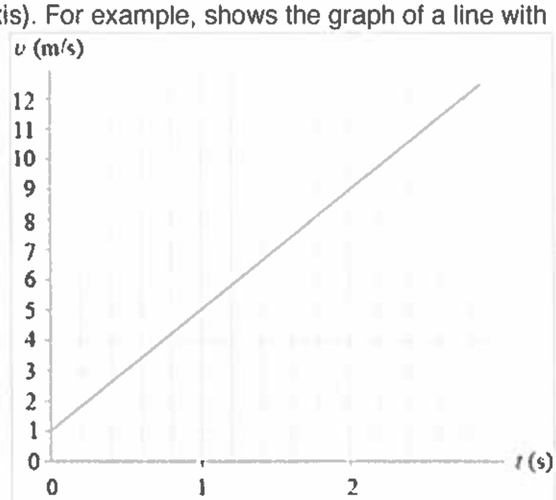
Before working on this primer, you may need to review:

- The slope-intercept form of a straight-line equation
- Finding the slope of a line
- The definitions for velocity and acceleration

For additional practice, you may want to review Finding the Equation (y=mx+b) from a Graph.

Interpreting the equation of a line

The generic slope-intercept format for writing the equation of a line is $y = mx + b$ where y is the parameter represented by the vertical axis on a graph, x is the parameter represented by the horizontal axis on the graph, m is the slope (i.e. rise/run) of the line and b is the y-intercept (i.e. the value where the graph crosses the vertical axis). For example, shows the graph of a line with the equation $v = 4t + 1$ where the vertical axis represents velocity (v) measured in $\mathrm{m/s}$ and the horizontal axis represents time (t) measured in s.

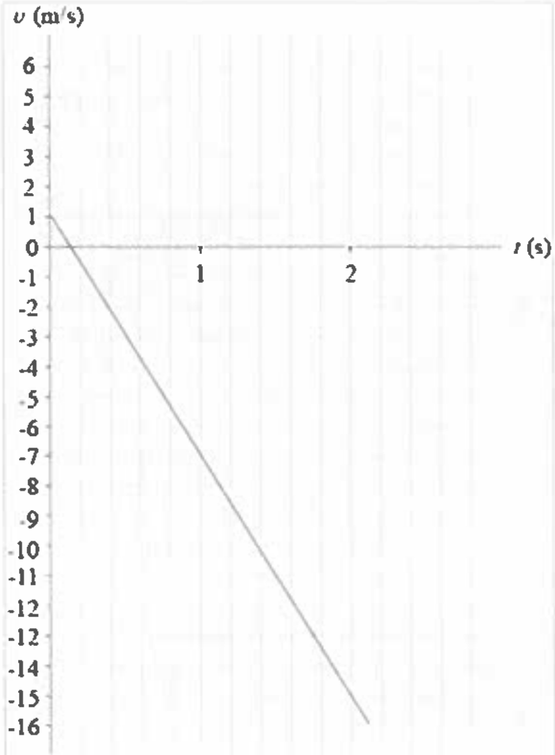


According to the generic slope-intercept format, the slope of the line is 4 and the value where the line crosses the vertical axis is 1. However, these values have physical meaning too. Since the units on the left side are $\mathrm{m/s}$, the terms on the right side must have units of $\mathrm{m/s}$ as well. If we plug in $t = 0\ \mathrm{s}$, we get $v = 1\ \mathrm{m/s}$, which tells us the velocity starts off at $1\ \mathrm{m/s}$ (also known as the initial velocity). Also, the slope must have units of $\frac{\mathrm{m/s}}{\mathrm{s}}$. This indicates that the slope of the line is $4\ \frac{\mathrm{m/s}}{\mathrm{s}}$, which represents the *acceleration*.

Effect of changing parameters

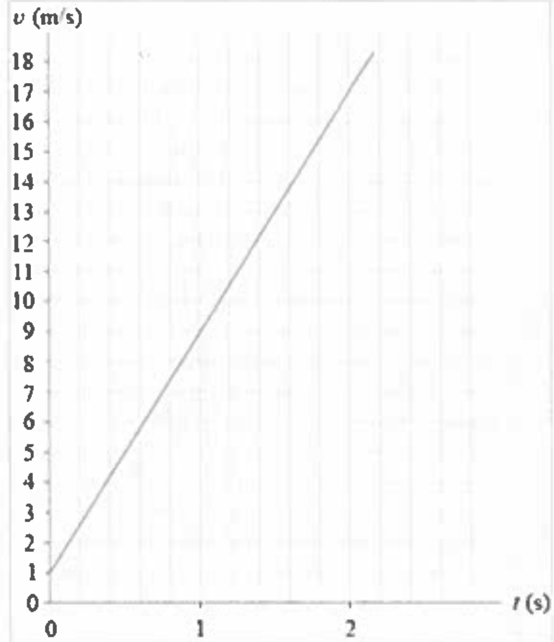
In , the line equation is $v = 8t + 1$. Here, the line is twice as steep, meaning the slope has doubled and the acceleration of the object with this motion is

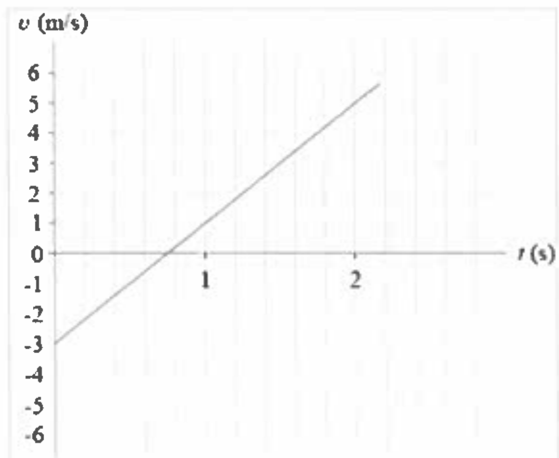
$8\ \frac{\text{m/s}}{\text{s}}$. The initial velocity on the other hand has not changed ($1\ \text{m/s}$).



In , the line equation is $v = -8t + 1$. Here, the line has the opposite slope, so the acceleration of the object with this motion is $-8\ \frac{\text{m/s}}{\text{s}}$. As before, the initial velocity has not changed ($1\ \text{m/s}$).

In , the line equation is $v = 4t - 3$. Here, the line shows the slope to be $4\ \frac{\text{m/s}}{\text{s}}$, while the initial velocity is now $-3\ \text{m/s}$.





Part A - Matching a graph to a line equation

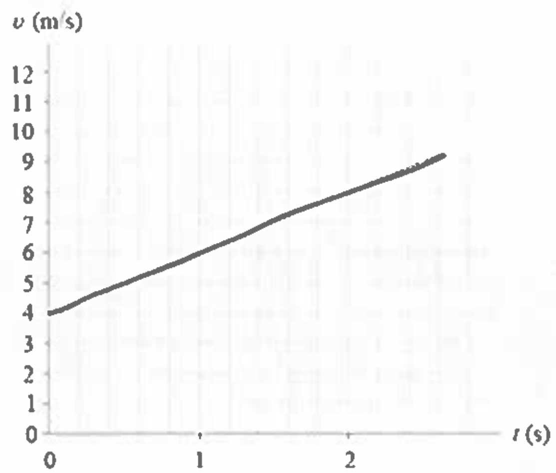
Which diagram shows the equation $v = 2t + 4$?

ANSWER:

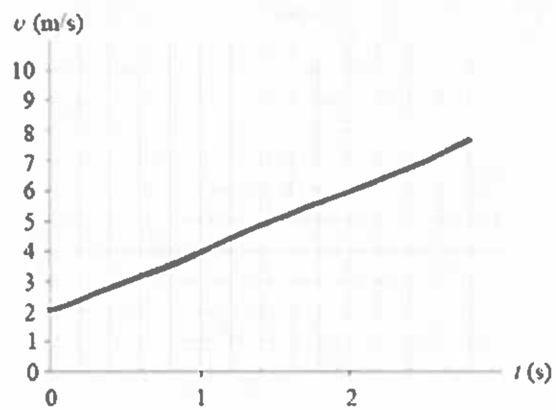
See next page.

22.

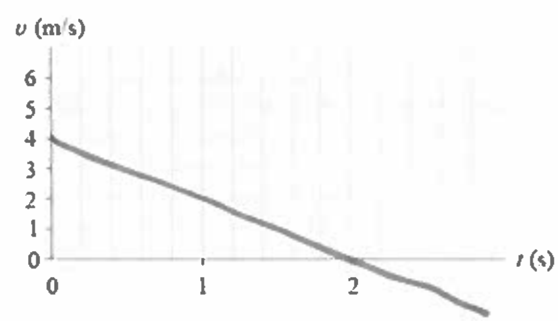
A



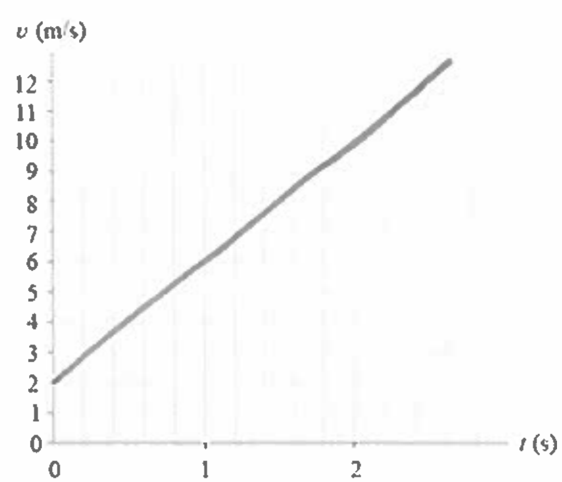
B



C



D



Part B - How changing the slope changes the graph

Which diagram shows the effect of cutting the acceleration in Part A in half and not changing the value of the initial velocity?

Hint 1. Which part of the equation represents slope

The acceleration is represented by the slope of the line.

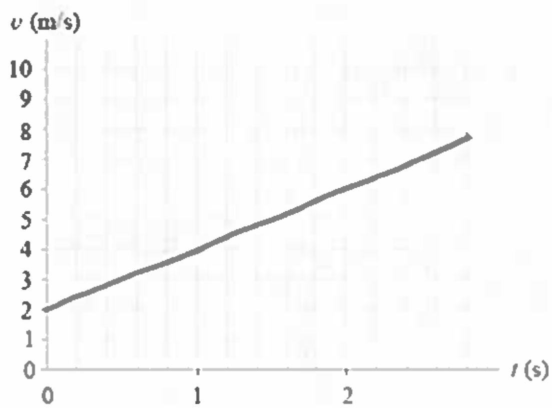
Hint 2. Relationship between slope and steepness

If the slope is cut in half, the graph should be half as steep.

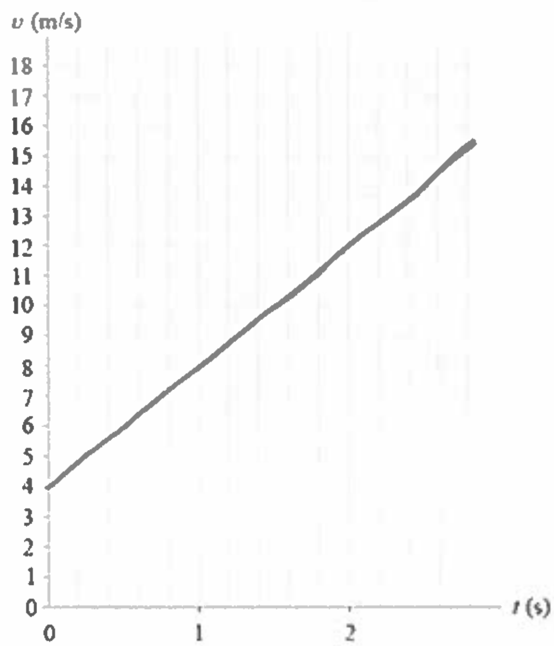
ANSWER:

23.

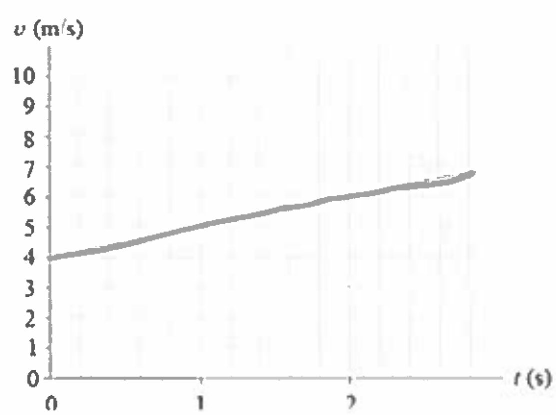
A

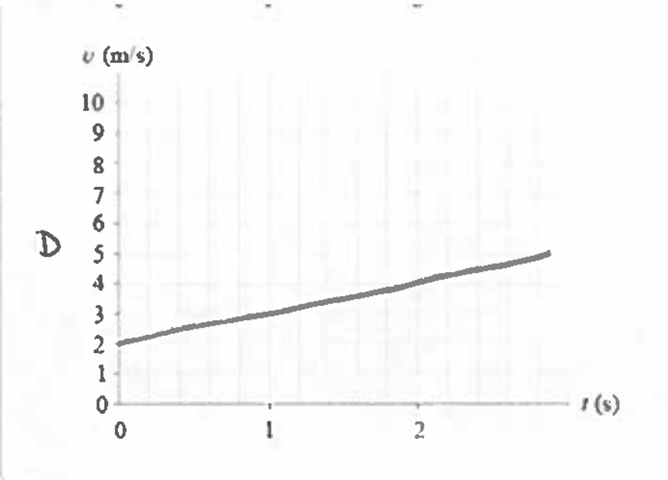


B



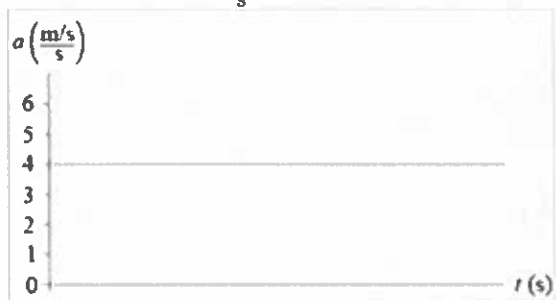
C



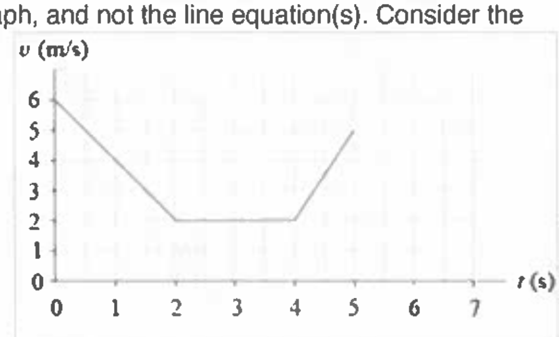


Interpreting the slope of a velocity vs. time graph

Figure 1 shows the graph of the equation $v = 4t + 1$. The slope of the line is a constant value of $4\ \frac{\text{m/s}}{\text{s}}$. This allows us to sketch an acceleration vs time graph as shown in . The equation of the line shown on this graph is $a = 4\ \frac{\text{m/s}}{\text{s}}$. In general, the slope of the velocity vs. time graph represents the acceleration.



We can use this slope relationship to determine the acceleration of an object if just given a velocity vs time graph, and not the line equation(s). Consider the velocity vs time graph in . This shows an object slowing down for two seconds, then moving at a constant rate for two seconds, then finally speeding up for one second.



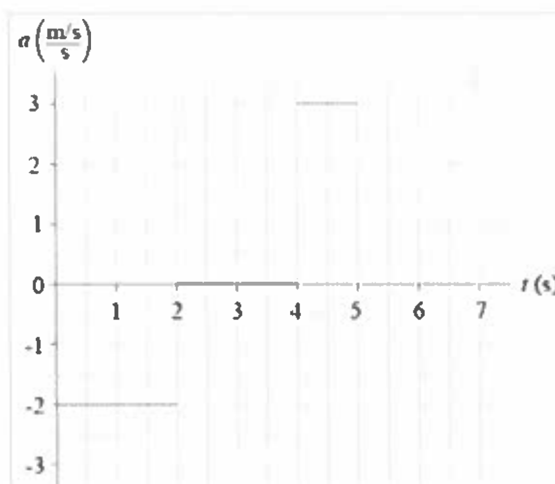
The slope of the first segment is $\frac{2\ \text{m/s} - 6\ \text{m/s}}{2\ \text{s}} = -2\ \frac{\text{m/s}}{\text{s}}$.

The slope of the second segment is $\frac{2\ \text{m/s} - 2\ \text{m/s}}{2\ \text{s}} = 0\ \frac{\text{m/s}}{\text{s}}$.

The slope of the third segment is $\frac{5\ \text{m/s} - 2\ \text{m/s}}{1\ \text{s}} = 3\ \frac{\text{m/s}}{\text{s}}$.

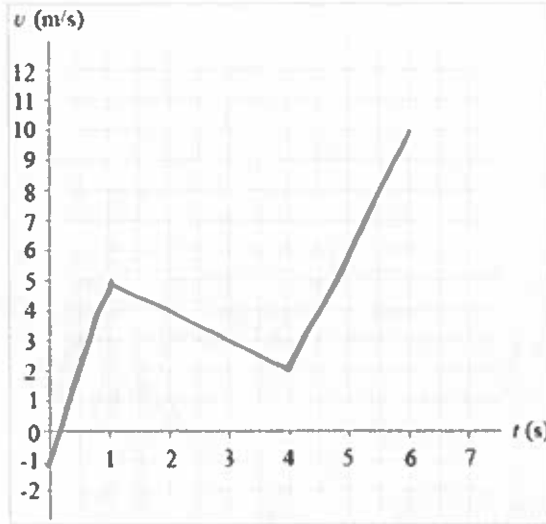
These can be represented on the acceleration graph shown in .

Note: Don't worry about the discontinuity between line segments. In reality, velocities can't change instantaneously so there would be a very steep line segment for a very short time interval between the flat segments.



Part C - Finding an acceleration graph from a velocity graph

Which is the correct acceleration vs. time graph for the velocity vs. time graph shown in ?



Hint 1. What part of the graph represents acceleration?

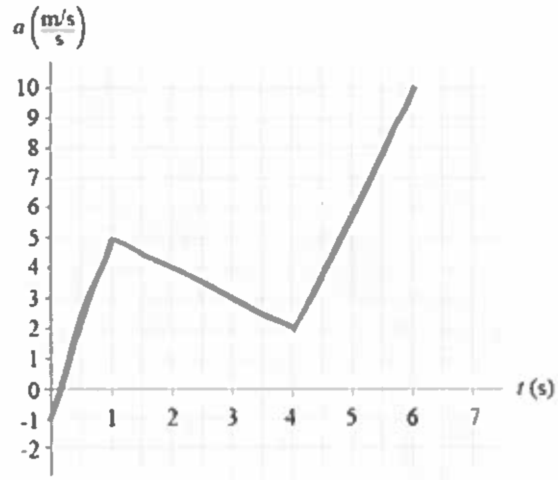
The acceleration is represented by the slope of the line.

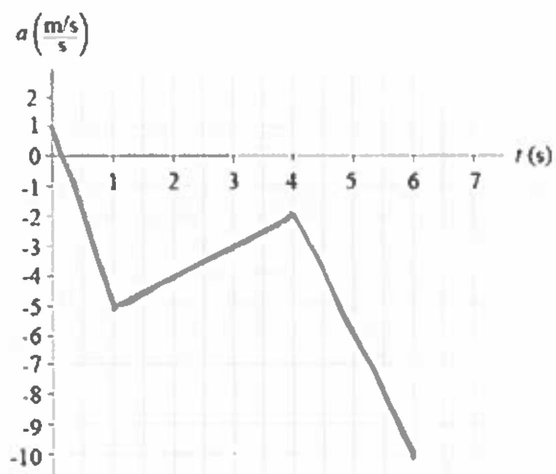
Hint 2. Finding the slope

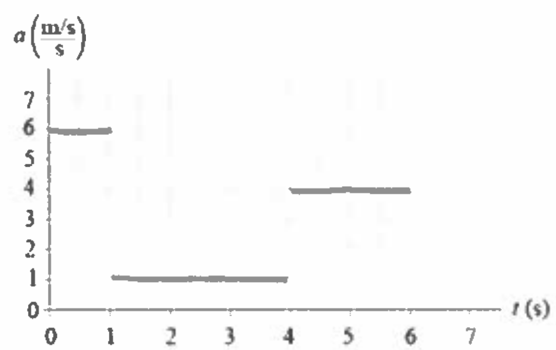
Find the slope of the line by calculating the rise of the line $v_2 - v_1$ divided by the run of the line $t_2 - t_1$.

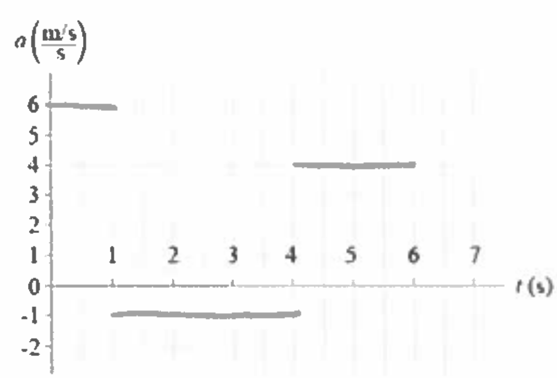
ANSWER:

24.



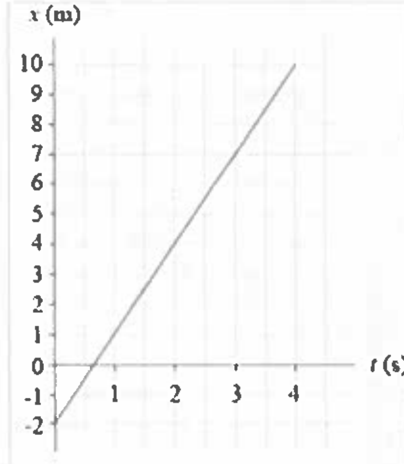






Interpreting the slope of a position vs. time graph

This same method can be used to determine the velocity vs. time graph from a simple position vs. time graph .

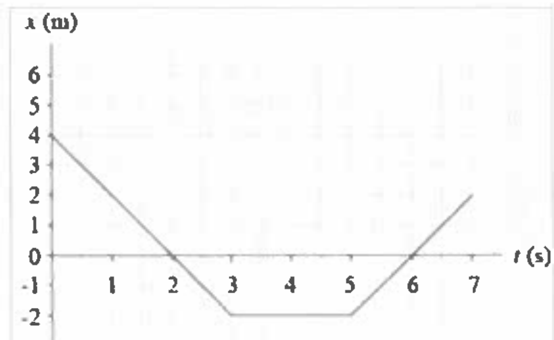


The equation for this graph is $x = 3t - 2$. If x is measured in meters, each term on the right side must be in meters as well. If we plug in $t = 0$ s, we get $x = -2$ m, which tells us the position starts off at -2 m (also known as the initial position). Also, the slope must have units of m/s. This indicates that the slope of the line is 3 m/s, which represents the velocity. This leads to the velocity vs. time graph shown in .

We can use this slope relationship to determine the velocity of an object if just given a position vs time graph, and not the line equation(s).

Part D - Finding a velocity graph from a position graph

Which is the correct velocity vs time graph for the position vs. time graph shown in ?



Hint 1. What part of the graph represents velocity?

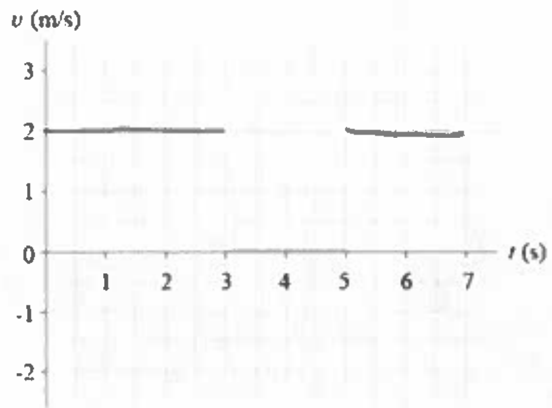
The velocity is represented by the slope of the line.

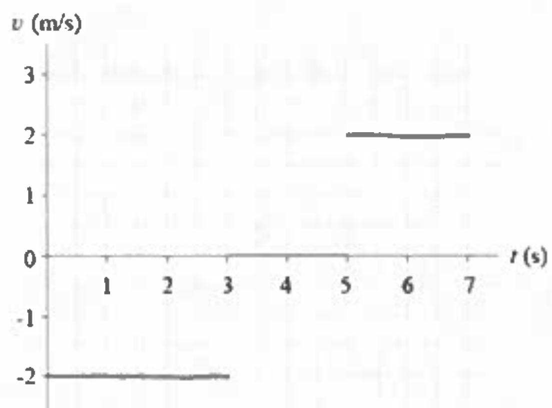
Hint 2. Finding the slope

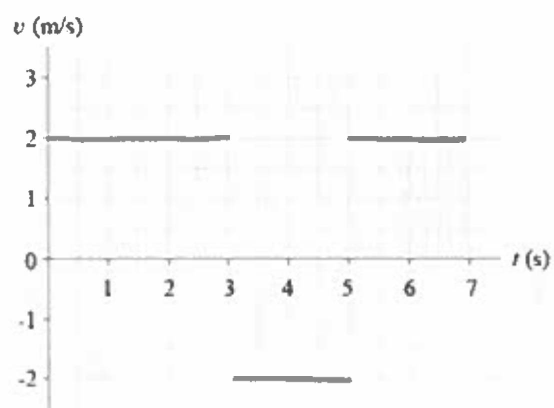
Find the slope of the line by calculating the rise of the line $x_2 - x_1$ divided by the run of the line $t_2 - t_1$.

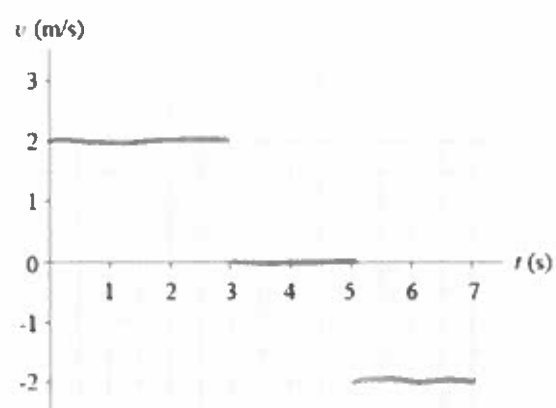
ANSWER:

25.









Right Triangles

Students who successfully complete this primer will be able to:

- Determine the length of a side of a right triangle, given the length of the two other sides.
- Determine the length of a side of a right triangle, given the length of one other side and an angle.

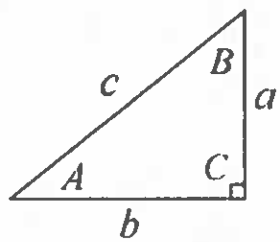
Before working on this primer, you may need to review:

- The meaning and application of the mnemonic "sohcahtoa".

For additional practice, you may want to review Right Triangle Calculations.

Introduction to Right Triangles

There are many applications of right triangles in physics and engineering such as surveying, construction, and even determining the distances to nearby stars. You also need a good understanding of right triangles in order to analyze vectors quantities in general (of which there is no shortage in physics!). Suppose you have a right triangle like the one depicted below



where side a is opposite angle A, side b is opposite angle B, and side c is opposite the right angle C, which is 90°. There are four primary relationships that are useful for solving the various parameters of such a right triangle, which are given by:

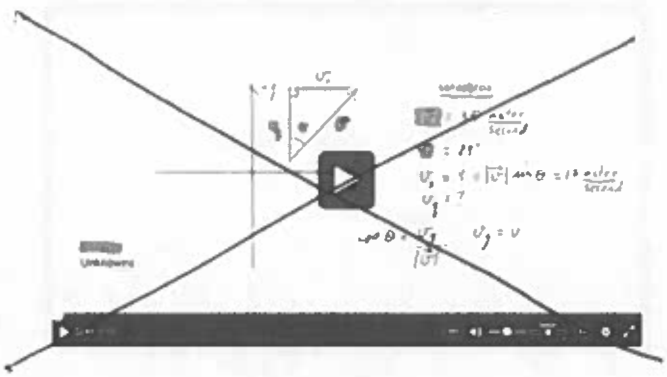
Pythagorean Theorem: $a^2 + b^2 = c^2$

Definition of sine: $\sin A = \frac{a}{c}$, $\sin B = \frac{b}{c}$
Definition of cosine: $\cos A = \frac{b}{c}$, $\cos B = \frac{a}{c}$
Definition of tangent: $\tan A = \frac{a}{b}$, $\tan B = \frac{b}{a}$

Which of these relationships you need to use depends on the specific parameter you are attempting to solve in a problem. When you know the length of two sides of a triangle but don't know the length of the third side, you should use the Pythagorean Theorem. When you know the length of one side of a triangle and the value of one angle (other than the right angle), you should use the corresponding trigonometric function. For example, if you knew the length of side c and the angle A, you could find the length of side a from the formula $\sin A = \frac{a}{c}$. In general, apply the formula that includes the two parameters you know as well as the one you don't know and intend to solve.

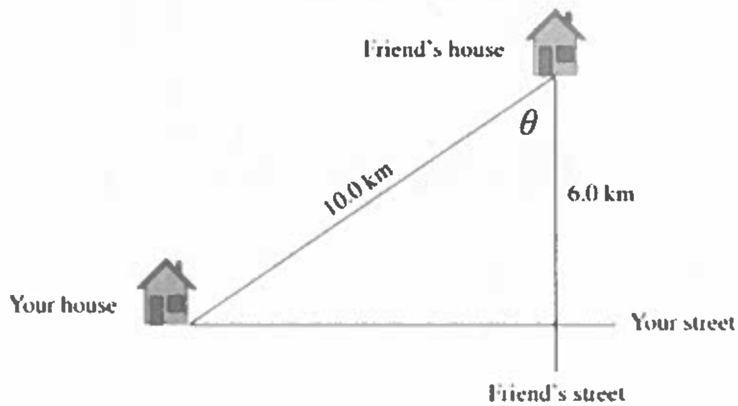
While we often think of right triangles as applying only to lengths, the rules listed above apply in general to any vector quantity, which can be depicted as having a magnitude (i.e. the hypotenuse of the triangle), and two components that are at right angles to one another and represent the other two sides of the triangle. Some examples include velocity (see Part C), force, and momentum.

~~Click here to watch a right triangles video, then answer the questions that follow.~~



Part A - Finding an unknown side when you know the two other sides

Your GPS shows that your friend's house is 10.0 km away, as shown in the image below. But there is a big hill between your houses and you don't want to bike there directly. You know your friend's street is 6.0 km north of your street. How far do you have to ride before turning north to get to your friend's house?



ANSWER:

26. km

Part B - Finding the value of a trigonometric function

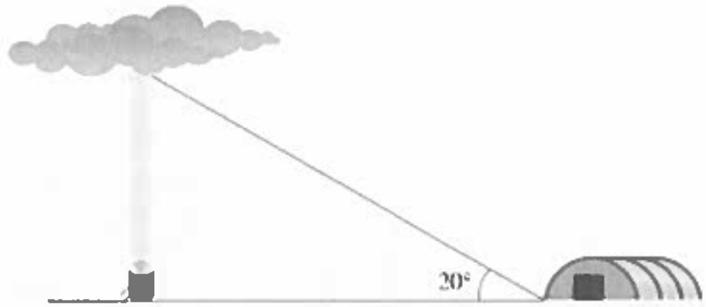
Referring to the diagram in Part A, what is the sine of the angle θ at the location of the friend's house?

ANSWER:

27. $\sin\theta =$

Part C - Finding an unknown side when you know one other side and an angle

In aviation, it is helpful for pilots to know the cloud ceiling, which is the distance between the ground and lowest cloud. The simplest way to measure this is by using a spotlight to shine a beam of light up at the clouds and measuring the angle between the ground and where the beam hits the clouds. If the spotlight on the ground is 0.75 km from the hangar door as shown in the image below, what is the cloud ceiling?

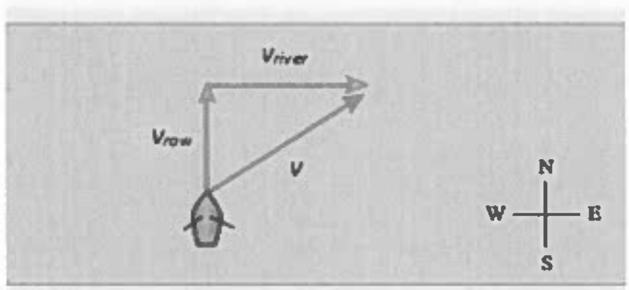


ANSWER:

28. km

Part D - Analyzing a right triangle consisting of something other than lengths

A person is rowing across the river with a velocity of 4.5 $\mathrm{km/hr}$ northward. The river is flowing eastward at 3.5 $\mathrm{km/hr}$ as shown in the image below. What is the magnitude of her velocity (v) with respect to the shore?



ANSWER:

29. $\mathrm{km/hr}$

Inverse Trigonometric Functions

Students who successfully complete this primer will be able to:

- Analyze physical quantities related through the properties of a right triangle to obtain expressions for angles in terms of inverse trigonometric functions.
- Use a calculator to evaluate the numerical value of angles using inverse trigonometric functions.

Before working on this primer, you may need to review:

- Trigonometric functions
- Using your calculator (specifically trigonometric functions)

For additional practice, you may want to review Right Triangle Calculations.

Overview of inverse trigonometric functions

1. Inverse trigonometric functions are used to obtain the value of an angle when the trigonometric function of that angle (e.g. $\cos(\theta)$, $\sin(\theta)$, or $\tan(\theta)$) is known. Knowing the value of $\sin(\theta)$ for instance is not the same as knowing the value of θ itself – that's the role played by inverse trigonometric functions.

2. The notation for inverse trigonometric functions is given in the following table:

If:	**Then:**
$\cos(\theta) = x$	$\theta = \arccos(x)$
$\sin(\theta) = x$	$\theta = \arcsin(x)$
$\tan(\theta) = x$	$\theta = \arctan(x)$

- Think of the "arc" as meaning: "the angle whose {cos, sin, tan} is..."
- Some textbooks and other materials (including most calculators) use the notation $\cos^{-1}(x)$, $\sin^{-1}(x)$, and $\tan^{-1}(x)$ for inverse trigonometric functions. When you encounter this notation, it is important to understand that it does ***not*** mean for instance $\frac{1}{\cos(x)}$ (in contrast to the use of the x^{-1} notation in other contexts)!

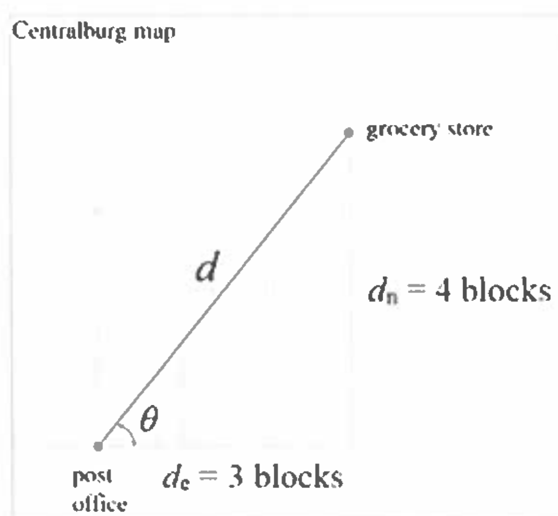
3. In a typical application of inverse trigonometric functions:

- Use the mnemonic ***SohCahToa*** to obtain an expression between a trigonometric function of an angle and physical quantities that correspond with the sides of a right triangle, and apply the appropriate inverse trigonometric function. For example, given $\tan(\theta) = \frac{d_y}{d_x}$ and knowing d_y and d_x with θ unknown, then $\theta = \arctan(\frac{d_y}{d_x})$.
- Use a calculator (be sure to set it to "degree" as the unit for angle!) to evaluate the value of an inverse trigonometric function.

With these properties in mind, answer the following questions.

Part A

In the town of Centralburg , which is laid out in a uniform block grid, the grocery store is three blocks East and four blocks North of the post office. Which of the following is a correct equation for the quantities represented in this scenario?



ANSWER:

30.

A $\tan(\theta) = \frac{d_n}{d_e}$

B $\theta = \frac{d_n}{d_e}$

C $\arctan(\theta) = \frac{d_n}{d_e}$

D $\sin(\theta) = \frac{d_n}{d_e}$

Part B

Which of the following is a correct equation for the angle θ?

ANSWER:

31.

A $\theta = \frac{d_n}{d}$

B $\theta = \arctan\left(\frac{d_n}{d_e}\right)$

C $\theta = \frac{d_e}{d}$

D $\theta = \arcsin\left(\frac{d_n}{d_e}\right)$

Part C

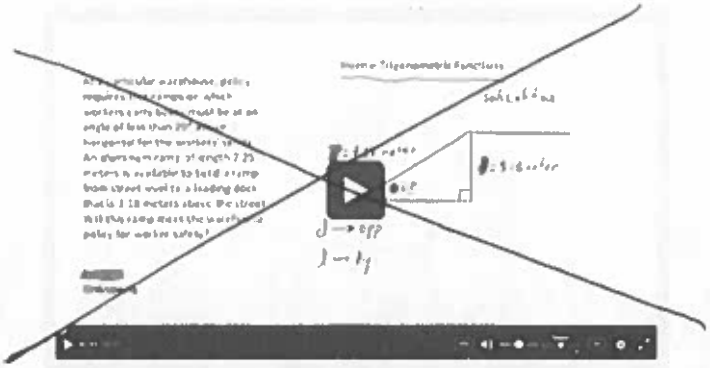
What is the angle (in degrees) North of East from the post office to the grocery store in Centralburg?

Express your answer to two significant figures.

ANSWER:

32. $\theta =$ $^\circ$

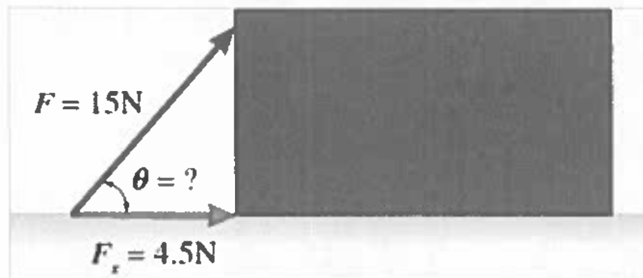
~~Click here to watch a video that walks through an example on inverse trigonometric functions, then answer the question that follows.~~



Part D

A force is applied to a block sliding along a surface . The magnitude of the force is 15 N, and the horizontal component of the force is 4.5 N. At what angle (in degrees) above the horizontal is the force directed?

Express your answer to two significant figures.



Hint 1. How to approach the problem

As shown in the diagram, the force and its x (horizontal) and y (vertical) components form the sides of a right triangle. That means you can use SohCahToa to write expressions relating them to trigonometric functions of angles.

ANSWER:

33. $\theta =$ $^\circ$

Vector Math

21 questions

Pythagorean Theorem

Learning Goal:
To understand and apply the Pythagorean Theorem.

The Pythagorean Theorem is named after a religious school from ancient Greece whose students believed whole numbers to be the foundations of the universe. They discovered much interesting math using whole numbers. However, the discovery that they are most famous for also led to the downfall of their religion! The Pythagorean Theorem leads directly to the discovery of irrational numbers—numbers that cannot be written as the ratio of two whole numbers. Seeing that even something as simple as the diagonal of a square leads to irrational numbers shattered their belief in the holiness of whole numbers, but this insight also laid the foundation for many of the discoveries that made Greek mathematics, particularly geometry, so successful.

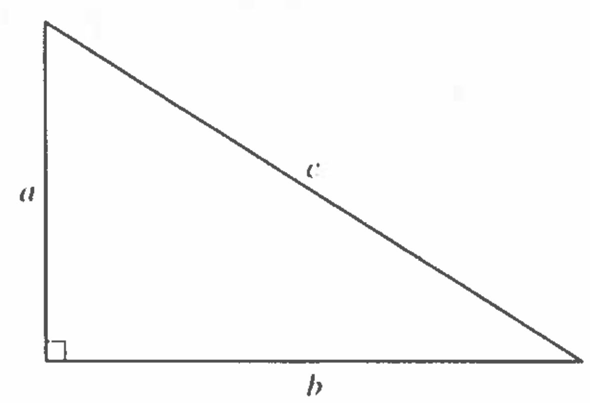
The Pythagorean Theorem relates the lengths of the two legs (the sides opposite the two acute angles) a and b of a right triangle to the length of the hypotenuse (the side opposite the right angle) c. Given a right triangle as shown in , the Pythagorean Theorem is written

$$a^2 + b^2 = c^2.$$

For instance, if you had a right triangle with legs both of length 1 (i.e., $a = b = 1$), then the Pythagorean Theorem would give

$$c^2 = 1^2 + 1^2 = 2,$$

so that $c = \sqrt{2}$.



Part A

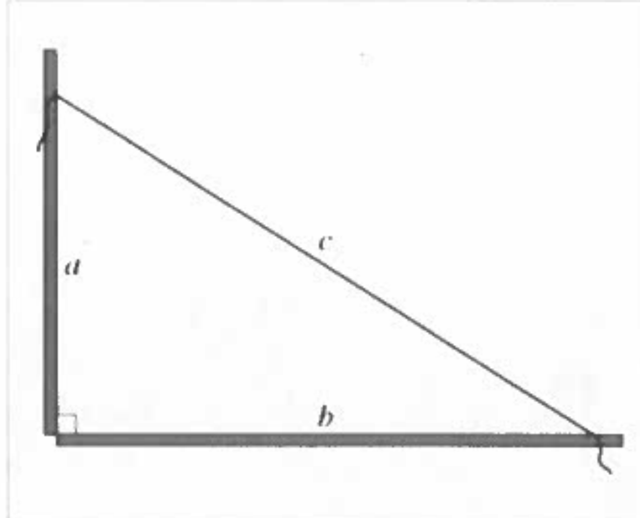
Now, consider a right triangle with legs of lengths 5 cm and 12 cm. What is the length c of the hypotenuse of this triangle?

Express your answer to three significant figures.

ANSWER:

1. c = cm

One application of the Pythagorean Theorem is in making sure that two boards are perpendicular (form a 90° angle) when doing construction, carpentry, etc. If you have two boards and measure out distances on each (marked in blue on), then you can adjust the angle between them until the length of the string is equal to the hypotenuse predicted by the Pythagorean Theorem. At that point, the angle between the boards must be a right angle, to the level of precision that you've measured the lengths.



Part B

Suppose that you have measured a length of 6 cm on one board and 8 cm on the other. You would adjust the two boards until the length of the string had value c to ensure that the boards made a right angle. What is c?

Express your answer in centimeters to three significant figures.

ANSWER:

2. c = cm

Part C

Use the Pythagorean Theorem to determine which of the following give the measures of the legs and hypotenuse of a right triangle.

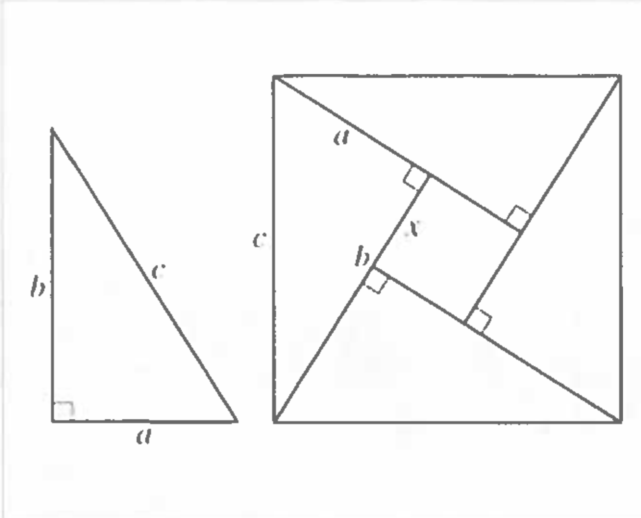
Check all that apply.

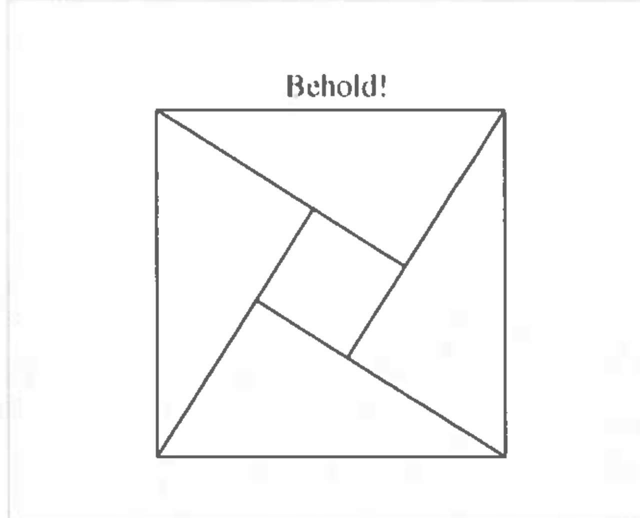
ANSWER:

3.
- A 3, 4, 5
- B 4, 11, 14
- C 9, 14, 17
- D 8, 14, 16
- E 8, 15, 17

Being one of the most famous theorems in all of geometry, many different proofs of the Pythagorean Theorem have been found. One of the most interesting was given by the Indian mathematician Bhaskara, who lived between 1114 and 1185. The entire proof consisted of a single cryptic image as shown in .

To try to understand this proof, let us consider the specific details in Bhaskara's drawing. makes it clear that the original square is made up of four identical right triangles and a smaller square.





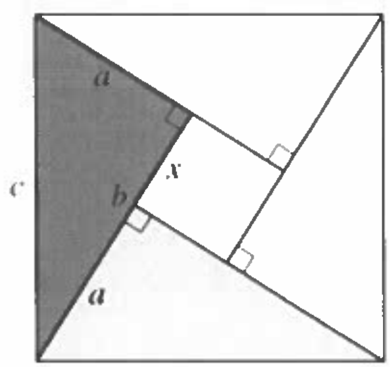
Part D

What is the length x of a side of the small inner square?

Express your answer in terms of the variables a and b.

Hint 1. A more helpful figure

In the figure below, the side of length a for the yellow triangle plus the side of the small square makes up the side of length b for the light blue triangle. You can set up a simple equation relating a, b, and x using this fact. Solve this equation for x.



ANSWER:

4. x =

Part E

Given that the side of the square has a length $b - a$, find the area of one of the four triangles and the area of the small inner square.

Give the area of one of the triangles followed by the area of the small inner square separated by a comma. Express your answers in terms of the variables a and b.

ANSWER:

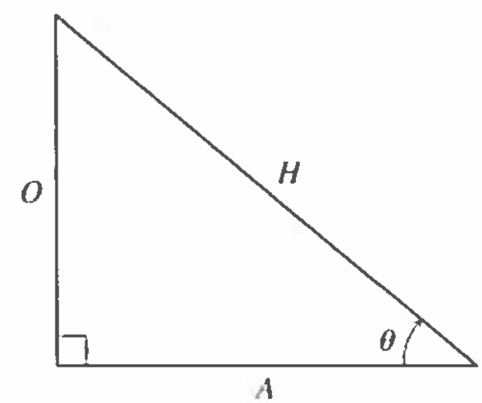
5. $A_{\text{triangle}}, A_{\text{square}}$ =

Trig Functions and Right Triangles

Learning Goal:
To use trigonometric functions to find sides and angles of right triangles.

The functions sine, cosine, and tangent are called *trigonometric* functions (often shortened to "trig functions"). Trigonometric just means "measuring triangles." These functions are called trigonometric because they are used to find the lengths of sides or the measures of angles for right triangles. They can be used, with some effort, to find measures of any triangle, but in this problem we will focus on right triangles. Right triangles are by far the most commonly used triangles in physics, and they are particularly easy to measure.

The sine, cosine, and tangent functions of an acute angle in a right triangle are defined using the relative labels "opposite side" O and "adjacent side" A. The hypotenuse H is the side opposite the right angle.



As you can see from the figure, the opposite side O is the side of the triangle not involved in making the angle. The side called the adjacent side A is the side involved in making the angle that is *not* the hypotenuse. (The hypotenuse will always be one of the two sides making up the angle, because you will always look at the acute angles, not the right angle.)

The sine function of an angle θ, written $\sin(\theta)$, is defined as the ratio of the length O of opposite side to the length H of the hypotenuse:

$$\sin(\theta) = \frac{O}{H}.$$

You can use your calculator to find the value of sine for any angle. You can then use the sine to find the length of the hypotenuse from the length of the opposite side, or vice versa, by using the fact that the previous formula may be rewritten in either of the following two forms:

$$O = H\sin(\theta)$$

or

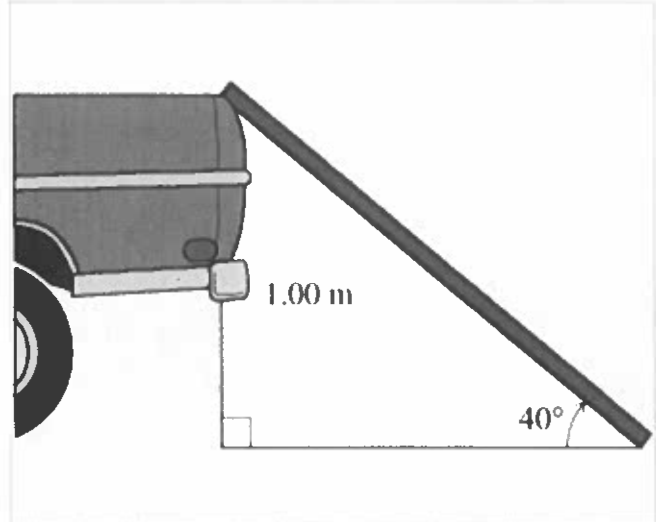
$$H = \frac{O}{\sin(\theta)}.$$

Part A

Suppose that you need to get a heavy couch into the bed of a pickup truck. You know the bed of the truck is at a height of 1.00 m and you need a ramp that makes an angle of 40° with the ground if you are to be able to push the couch.

Use the sine function to determine how long of a board you need to use to make a ramp that just reaches the 1.00-m high truck bed at a 40° angle to the ground.

Express your answer in meters to three significant figures.



ANSWER:

6. m

The cosine function is another useful trig function. The definition of the cosine function is similar to the definition of the sine function:

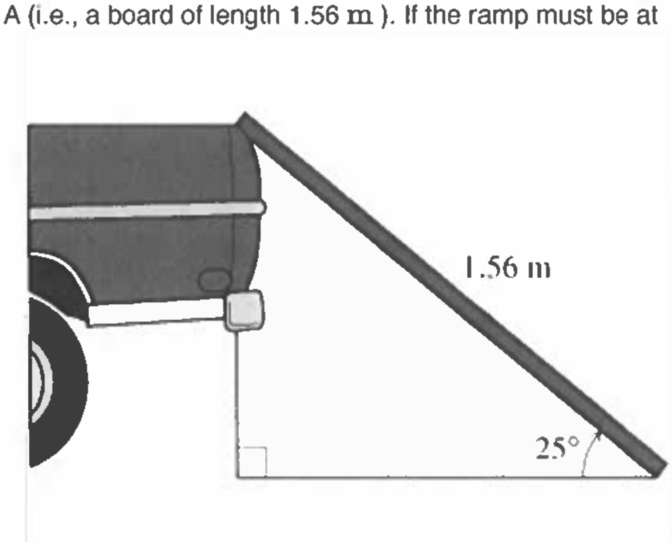
$$\cos(\theta) = \frac{A}{H}.$$

This equation can be rearranged the same way that the equation for sine was rearranged. With the cosine of an angle, you can find the length of the adjacent side from the length of the hypotenuse, or vice versa.

Part B

You need to set up another simple ramp using the board from Part A (i.e., a board of length 1.56 m). If the ramp must be at a 25° angle above the ground, how far back from the bed of the truck should the board touch the ground? Assume this is a different truck than the one from Part A.

Express your answer in meters to three significant figures.



Hint 1. Using the cosine function

The ramp is the *hypotenuse* of the right triangle in the figure, and the distance along the ground is *adjacent* to the 25° angle. To find the length of the adjacent side, use the

$$A = H\cos(\theta)$$

form of the cosine formula. Plugging in the given values will give you the distance along the ground.

ANSWER:

7. m

The third frequently used trig function is the tangent function. The tangent of an angle θ is defined by the equation

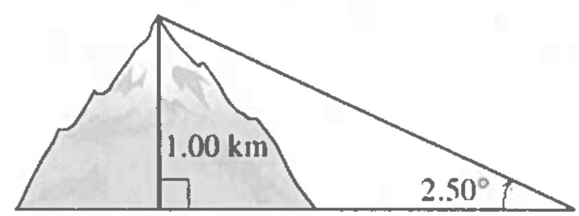
$$\tan(\theta) = \frac{O}{A}.$$

This equation can be rearranged the same way that the equations for sine and cosine were rearranged previously. With the tangent of an angle, you can find the length of the adjacent side from the length of the opposite side or vice versa.

Part C

Surveyors frequently use trig functions. Suppose that you measure the angle from your position to the top of a mountain to be 2.50°. If the mountain is 1.00 km higher in elevation than your position, how far away is the mountain?

Express your answer in kilometers to three significant figures.



Hint 1. Using the tangent function

The height of the mountain is *opposite* the 2.50° angle of the right triangle in the figure, and the distance to the mountain is *adjacent* to the 2.50° angle. To find the distance to the mountain, use the

$$A = \frac{O}{\tan(\theta)}$$

form of the tangent formula. Plugging in the given values will give you the distance to the mountain.

ANSWER:

8. km

All of the trig functions also have inverses. The inverses of the sine, cosine, and tangent functions are written as $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$, respectively. [Be careful not to confuse the notation $\sin^{-1}(x)$ for the inverse sine function with $(\sin(x))^{-1} = 1/\sin(x)$.] These inverse functions are also sometimes written asin, acos, and atan, short for arcsine, arccosine, and arctangent, respectively. Your calculator should have three buttons with one of those sets of three labels.

Since a trig function takes an angle and gives a ratio of sides, the inverse trig functions must take as input a ratio of sides and then give back an angle. For example, if you know that the length of the side adjacent to a particular angle θ is 12 cm and the length of the hypotenuse of this triangle is 13 cm, you can find the measure of angle θ using the inverse cosine. The cosine of θ would be 12/13, so the inverse cosine of 12/13 will give the value of θ:

$$\cos(\theta) = \frac{12}{13}$$

implies that

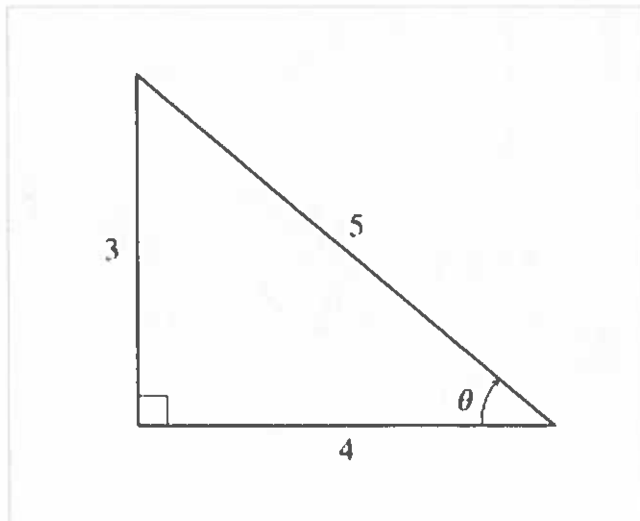
$$\theta = \mathrm{acos}\left(\frac{12}{13}\right).$$

Using the $\cos^{-1}$ or acos button on your calculator, you should check that the measure of θ is 22.6°.

Part D

The 3-4-5 right triangle is a commonly used right triangle. Use the inverse sine function to determine the measure of the angle opposite the side of length 3.

Express your angle in degrees to three significant figures.



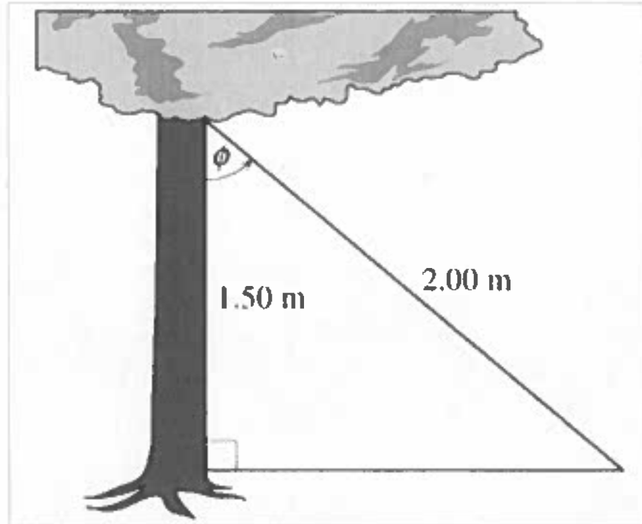
ANSWER:

9. θ = degrees

Part E

A support wire is attached to a recently transplanted tree to be sure that it stays vertical. The wire is attached to the tree at a point 1.50 m from the ground, and the wire is 2.00 m long. What is the angle ϕ between the tree and the support wire?

Express your answer in degrees to three significant figures.



ANSWER:

10. $\phi =$ degrees

Understanding Vector Addition

Learning Goal:
To learn to add vectors.

In physics, many important quantities—from the simple foundations of mechanics such as position and force to very foreign ideas such as electric current densities and magnetic fields—are vectors. Simply knowing what a vector is helps understanding, but to really use the ideas of physics and predict things, you need to be able to do calculations with those vectors. The simplest operation you might need to perform on vectors is to add them.

Suppose that you are swimming in a river while a friend watches from the shore. In calm water, you swim at a speed of 1.25 $\mathrm{m/s}$. The river has a current that runs at a speed of 1.00 $\mathrm{m/s}$.

Note that speed is the magnitude of the velocity vector. The velocity vector tells you both how fast something is moving and in which direction it is moving.

Part A

If you are swimming upstream (i.e., *against* the current), at what speed does your friend on the shore see you moving?

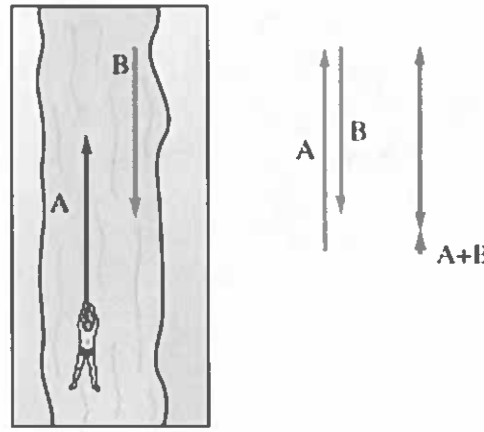
Express your answer in meters per second.

ANSWER:

11. $\mathrm{m/s}$

You likely could answer the last question without thinking about vectors at all. If a person swims against a current, it slows the person down. The speeds subtract in this case, because you are not actually adding speeds. You are adding velocities, which are vectors.

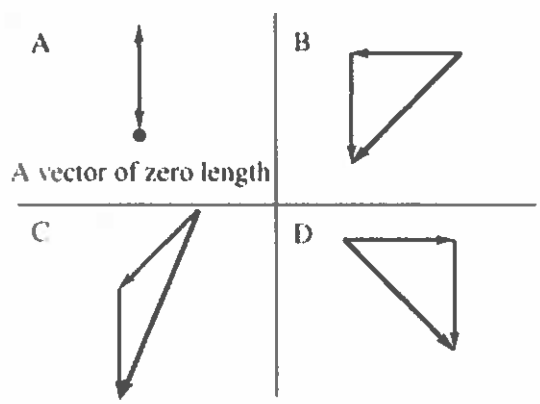
To add two vectors, say $\vec{A}+\vec{B}$, think of taking one vector ($\vec{B}$) and putting its tail on the head of the other vector ($\vec{A}$). The sum of the two vectors is then the vector that begins at the tail of $\vec{A}$ and ends at the head of $\vec{B}$.



Part B

If instead of swimming against the current you swam directly *across* the river (by your reckoning) at a speed of 1.00 $\mathrm{m/s}$ from left to right, which figure correctly shows the velocity vector with which your friend on the shore would see you moving?

Choose the correct figure. The added vectors are shown in gray; the vector representing their sum is shown in black.



ANSWER:

12.

- A
- B
- C
- D

Although drawing vectors is helpful for visualizing what happens when you add vectors, it is not a convenient way to calculate precise results: Adding components is preferable. When you add two vectors, the resulting vector's components are the sums of the corresponding components of the original vectors.

For instance, consider the two vectors $\vec{A}$ and $\vec{B}$ with components (a_x, a_y) and (b_x, b_y), respectively. If you want to find the sum $\vec{A}+\vec{B}$, then you would simply add the *x* components to get the resulting *x* component and add the *y* components to get the resulting *y* component:

$$\vec{A}+\vec{B}=(a_x+b_x, a_y+b_y).$$

For the situation of you swimming *across* the river from left to right at 1.00 $\mathrm{m/s}$, use a standard set of coordinates where the *x* axis is horizontal, with positive pointing to the right, and the *y* axis is vertical, with positive pointing upward.

Part C

Which of the following gives the correct components for the current velocity and the pure swimming velocity (i.e., the velocity that you would have in still water) using this coordinate system?

ANSWER:

A current: $(1.00\ \mathrm{m/s}, 0.00\ \mathrm{m/s})$; swimming: $(0.00\ \mathrm{m/s}, 1.00\ \mathrm{m/s})$

B current: $(0.00\ \mathrm{m/s}, 1.00\ \mathrm{m/s})$; swimming: $(1.00\ \mathrm{m/s}, 0.00\ \mathrm{m/s})$

C current: $(-1.00\ \mathrm{m/s}, 0.00\ \mathrm{m/s})$; swimming: $(0.00\ \mathrm{m/s}, -1.00\ \mathrm{m/s})$

D current: $(0.00\ \mathrm{m/s}, 1.00\ \mathrm{m/s})$; swimming: $(-1.00\ \mathrm{m/s}, 0.00\ \mathrm{m/s})$

13 E current: $(0.00\ \mathrm{m/s}, -1.00\ \mathrm{m/s})$; swimming: $(1.00\ \mathrm{m/s}, 0.00\ \mathrm{m/s})$

Part D

What is the resultant velocity vector when you add your swimming velocity and the current velocity?

Give the *x* and *y* components in meters per second separated by a comma.

ANSWER:

14. $\mathrm{m/s}$

Part E

Consider the two vectors $\vec{C}$ and $\vec{D}$, defined as follows:

$$\vec{C}=(2.35, -4.27) \text{ and } \vec{D}=(-1.30, -2.21).$$

What is the resultant vector $\vec{R}=\vec{C}+\vec{D}$?

Give the *x* and *y* components of $\vec{R}$ separated by a comma.

ANSWER:

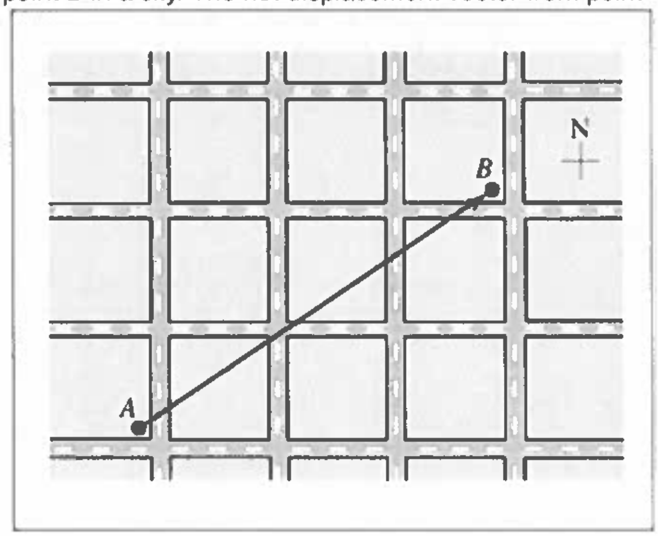
15. R_x, R_y =

Understanding Components of Vectors

Learning Goal:
To understand and be able to calculate the components of vectors.

You have heard vectors defined as quantities with magnitude and direction, familiar ideas also found in statements such as "three miles northeast of here." Components, the lengths in the *x* and *y* directions of the vector, are a different way to define vectors. In this problem, you will learn about components, by considering ways that they arise in everyday life.

Suppose that you needed to tell some friends how to get from point A to point B in a city. The net displacement vector from point A to point B is shown in the figure. You could tell them that to get from A to B they should go 3.606 blocks in a direction 33.69° north of east. However, these instructions would be difficult to follow, considering the buildings in the way.



Part A

You would more likely give your friends a number of blocks to go east and then a number of blocks to go north. What would these two numbers be?

Enter the number of blocks to go east, followed by the number of blocks to go north, separated by a comma.

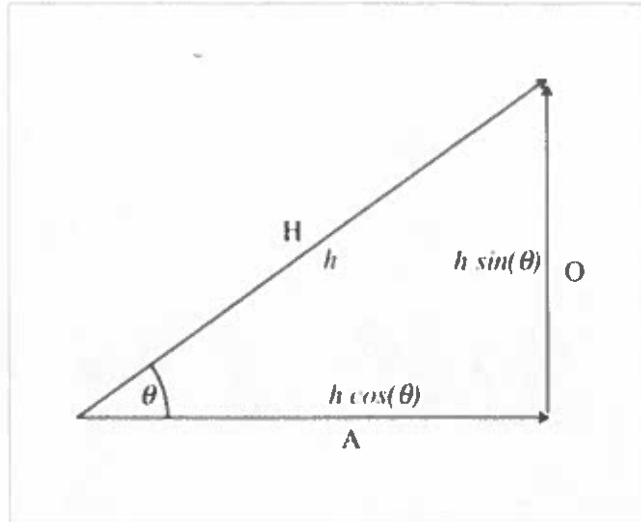
ANSWER:

16. blocks

Notice that the figure with the component vectors drawn in has the shape of a right triangle. You can use trigonometry to find the components of any vector.

Recall that, for some angle θ in a right triangle, the sine of that angle, $\sin(\theta)$, is defined as the length of the side (O) opposite the angle divided by the length of the hypotenuse (H) of the triangle, and the cosine of the angle, $\cos(\theta)$, is defined as the length of the side (A) adjacent to the angle divided by the length of the hypotenuse of the triangle.

In terms of these definitions, and the hypotenuse's length h, the triangle's sides have the following lengths: $h\sin(\theta)$ for the side opposite the angle, and $h\cos(\theta)$ for the side adjacent to the angle.



Part B

Consider the vector $\vec{b}$ with magnitude 4.00 m at an angle 23.5° north of east. What is the *x* component b_x of this vector?

Express your answer in meters to three significant figures.

ANSWER:

7. $b_x =$ m

Part C

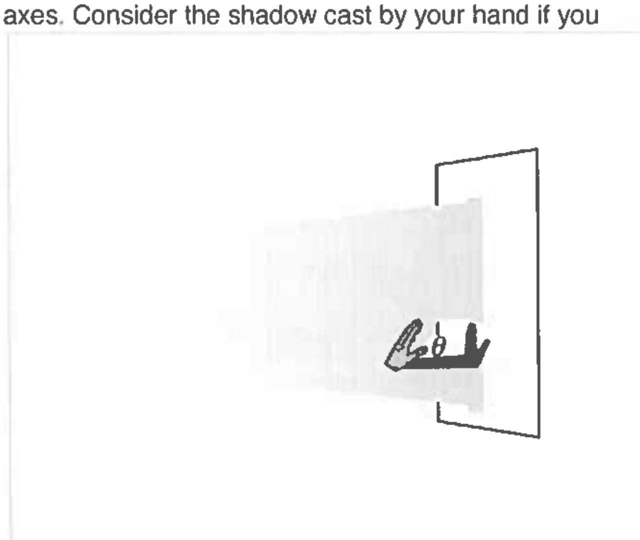
Consider the vector $\vec{b}$ with length 4.00 m at an angle 23.5° north of east. What is the *y* component b_y of this vector?

Express your answer in meters to three significant figures.

ANSWER:

18. $b_y =$ m

You can also think about components as projections onto the coordinate axes. Consider the shadow cast by your hand if you hold it near a movie screen. As you tilt your hand closer to the horizontal, the shadow gets smaller. If you think of your hand as a vector with tail at the base of your palm and arrow at your fingertips, the shadow's height corresponds to the *y* component of the vector. If you were to shine another light from above, the shadow cast below your hand would correspond to the *x* component.



Part D

What is the length of the shadow cast on the vertical screen by your 10.0 cm hand if it is held at an angle of $\theta = 30.0°$ above horizontal?

Express your answer in centimeters to three significant figures.

Hint 1. Which component do you need?

Since the shadow is vertical and the horizontal direction coincides with the positive *x* axis, the shadow of your hand would be the *y* component of the "hand vector."

ANSWER:

19. | cm |

You can also use your knowledge of right triangles to solve the problem in reverse, that is, to find the magnitude and direction of a vector from its components.

If you know the two components of a two-dimensional vector, you can use the Pythagorean Theorem to find the vector's magnitude (i.e., length) by adding the squares of the two components and then taking the square root. In Parts B and C, the two components were 3.668 m and 1.595 m , and the vector's magnitude is

$$|\vec{v}| = \sqrt{(3.668)^2 + (1.595)^2} = 4.000.$$

Part E

What is the magnitude of a vector with components (15 m, 8 m)?

Express your answer in meters.

Hint 1. More about the Pythagorean Theorem

Recall from geometry that the Pythagorean Theorem says

$$a^2 + b^2 = c^2,$$

where a and b are the lengths of the two legs of a right triangle and c is the length of the hypotenuse. You know from the previous discussion that the *x* and *y* components of a vector can be thought of as the lengths of two legs of a right triangle with the vector itself as hypotenuse. Therefore the magnitude of the vector $|\vec{v}|$ and the two components v_x and v_y must satisfy the Pythagorean Theorem:

$$v_x^2 + v_y^2 = v^2.$$

Taking the square root of both sides gives the relation

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}.$$

ANSWER:

20. | m |

Finding the direction from the components requires a bit of trigonometry. In a right triangle, the tangent of an angle is the length of the side (O) opposite the angle divided by the length of the side (A) adjacent to the angle.

Using this definition and the figure showing the right triangle, you can see that the tangent of the angle above the positive *x* axis is the *y* component of the vector (the length of side O) divided by the *x* component of the vector (the length of side A):

$$\tan(\theta) = \frac{v_y}{v_x}.$$

When you use this formula, remember that you are finding the angle measured counterclockwise from the positive *x* axis. Sometimes you will be asked for the angle with other axes. You should be able to use the same trigonometry described here, but this formula may not be quite right.

Part F

What is the angle above the *x* axis (i.e., "north of east") for a vector with components ($15\ \mathrm{m}$, $8\ \mathrm{m}$)?

Express your answer in degrees to three significant figures.

ANSWER:

21. degrees

 13 questions

Proportional Reasoning

Learning Goal:
To understand proportional reasoning for solving and checking problems.

Proportional reasoning involves the ability to understand and compare ratios and to produce equivalent ratios. It is is a very powerful tool in physics and can be used for solving many problems. It's also an excellent way to check answers to most problems you'll encounter. Proportional reasoning is something you may already do instinctively without realizing it.

Part A

You are asked to bake muffins for a breakfast meeting. Just as you are about to start making them, you get a call saying that the number of people coming to the meeting has doubled! Your original recipe called for three eggs.How many eggs do you need to make twice as many muffins?

Express your answer as an integer.

ANSWER:

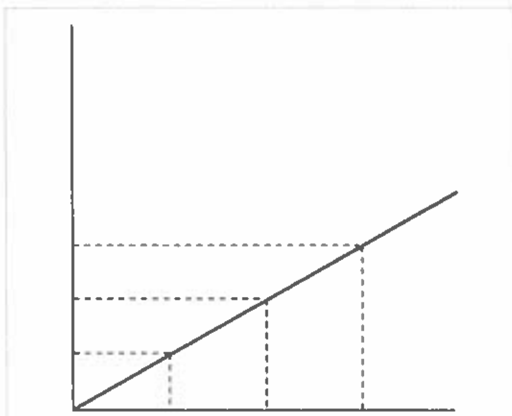
1.

Linear relationships

Although this was a particularly simple example, you used proportional reasoning to solve this problem. It makes sense that if you need twice as many muffins, then you'd need twice as many eggs to make them. We say that the number of eggs is *linearly proportional* to the number of muffins. This sort of relationship is characterized by an equation of the form $y = kx$, where y and x are the two quantities being related (eggs and muffins here) and k is some constant. In a situation where the constant k is not important, we may write $y \propto x$, which means "y is proportional to x".

Writing $(\text{number of muffins}) \propto (\text{number of eggs})$ means we know that if the number of eggs triples, then the number of muffins triples as well. Or, if the number of muffins is divided by 5, then the number of eggs is divided by 5.

The figure shows a graph of $y = kx$ for some constant k. You can see that when you double or triple the original x value, you get double or triple the y value, respectively. Keep this graph in mind and relate it to your intuitive sense as you solve the next problem.



Part B

You have a dozen eggs at home, and you know that with them you can make 100 muffins. If you find that half of the eggs have gone bad and can't be used, how many muffins can you make?

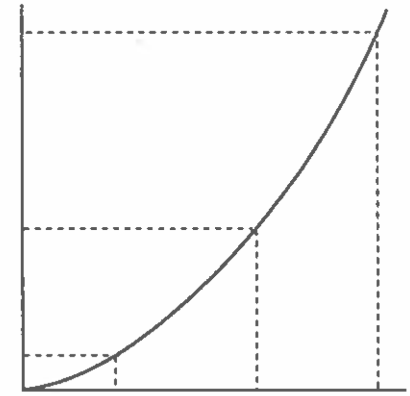
Express your answer as an integer.

ANSWER:

2.

Quadratic relationships

Quadratic relationships are also important in physics and many other areas. In a quadratic relationship, if one number is increased by a factor of a, then the other is increased by a factor of a^2. An example would be the relationship between area and radius of a circle. You know from geometry that $A = \pi r^2$. Since π is a constant, you can rewrite this equation as $A \propto r^2$, which says that A is proportional to *the square of* r. The relation $y \propto x^2$ applies to any equation of the form $y = kx^2$. The figure shows a graph of $y = kx^2$ for some constant k. You can see that when you double or triple the original x value, you get four or nine times the y value, respectively.



Part C

When sizes of pizzas are quoted in inches, the number quoted is the diameter of the pizza. A restaurant advertises an 8-inch "personal pizza." If this 8-inch pizza is the right size for one person, how many people can be fed by a large 16-inch pizza?

Express your answer numerically.

Hint 1. How to approach the problem

The area of a pizza is what determines how many people can be fed by the pizza. You know that the area of a circle is proportional to the square of the radius. Since the radius is proportional to the diameter, it follows that the area is also proportional to the square of the diameter: $A \propto d^2$. Use this relation to determine how the area, and therefore the number of people fed, changes.

ANSWER:

3.

The stopping distance is how far you move down the road in a car from the time you apply the brakes until the car stops. Stopping distance D is proportional to the square of the initial speed v at which you are driving: $D \propto v^2$.

Part D

If a car is speeding down a road at 40 $\mathrm{miles/hour}$ (mph), how long is the stopping distance D_{40} compared to the stopping distance D_{25} if the driver were going at the posted speed limit of 25 mph?

Express your answer as a multiple of the stopping distance at 25 mph. Note that D_{25} is already written for you, so just enter the number.

Hint 1. Setting up the ratio

Since $40/25 = 1.6$, the car is moving at a speed 1.6 times the speed limit of 25 mph. The stopping distance is proportional to the square of the initial speed, so the stopping distance will increase by a factor of the square of 1.6.

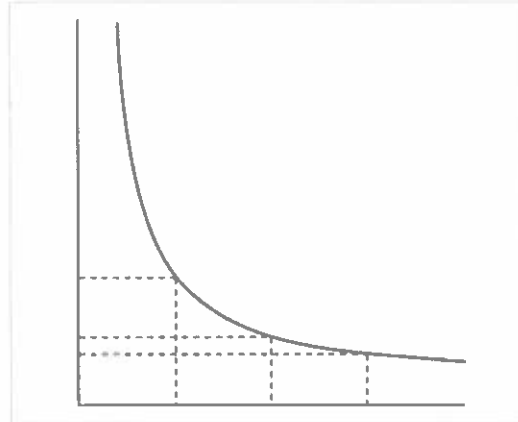
ANSWER:

4. D_{40} = $\times D_{25}$

Inverse relationships

A third important type of proportional relationship is the inverse relationship. In an inverse relationship, as one variable increases the other decreases and vice versa. For instance, if you had a $10 gift certificate to a chocolate shop, then the amount of chocolate that you could get would be inversely proportional to the price of the chocolate you picked. If you buy the $0.25 candies, you could get 40 of them, but if you opt to purchase candies whose price is higher by a factor of 4 ($1.00), then you must reduce the number that you get by a factor of 4 (to 10). Similarly, if the price decreases by a factor of 5 (to $0.05), then you increase the number by a factor of 5 (to 200).
An inverse relationship is based on an equation of the form $y = k/x$, where k is a constant. If y is inversely proportional to x then you would write $y \propto 1/x$ or $y \propto x^{-1}$.

The figure shows a graph of $y = k/x$ for some constant k. You can see that when you double or triple the original x value, you get one-half or one-third times the y value, respectively.



Part E

A construction team gives an estimate of three months to repave a large stretch of a very busy road. The government responds that it's too much inconvenience to have this busy road obstructed for three months, so the job must be completed in one month. How does this deadline change the number of workers needed?

Hint 1. The proportionality

The time to complete the job should be inversely proportional to the number of workers on the job. Therefore, *reducing* the time by a factor of 3 requires *increasing* the number of workers by a factor of 3.

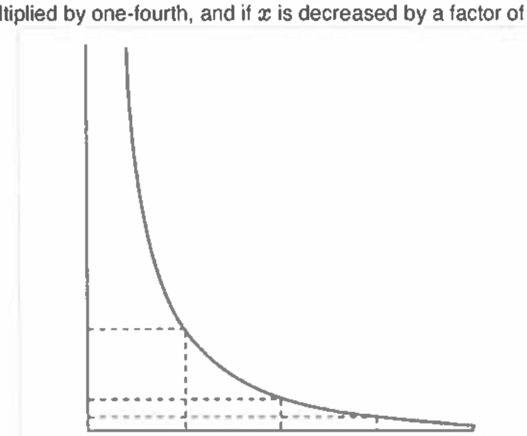
ANSWER:

5.

- A One-ninth as many workers are needed.
- B One-third as many workers are needed.
- C The same number of workers are needed.
- D Three times as many workers are needed.
- E Nine times as many workers are needed.

Inverse-square relationships

All of these proportionalities are in some way familiar to you in your everyday life. There is one other important type in physics with which you may not be as familiar: the inverse-square relationship. The inverse-square relationship is based on an equation of the form $y = k/x^2$, where k is a constant. You would write $y \propto 1/x^2$ or $y \propto x^{-2}$, either of which means "y is inversely proportional to the square of x." Although this may look or sound more intimidating than the relations we've looked at previously, it works in essentially the same way. If x is doubled, then y is multiplied by one-fourth, and if x is decreased by a factor of 2, y is multiplied by 4. The figure shows a graph of $y = k/x^2$ for some k. You can see that when x *increases* by a factor of 2 or 3, y *decreases* by a factor of 4 or 9, respectively.



Part F

The loudness of a sound is inversely proportional to the square of your distance from the source of the sound. If your friend is right next to the speakers at a loud concert and you are four times as far away from the speakers, how does the loudness of the music at your position compare to the loudness at your friend's position?

ANSWER:

6.

A The sound is one-sixteenth as loud at your position.

B The sound is one-fourth as loud at your position.

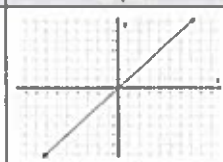
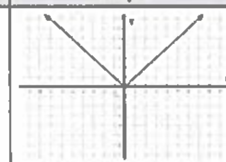
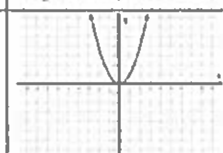
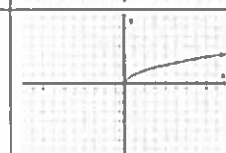
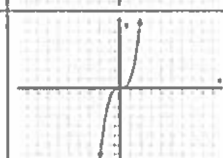
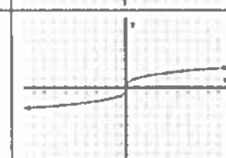
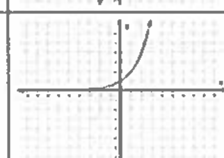
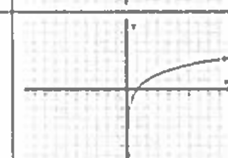
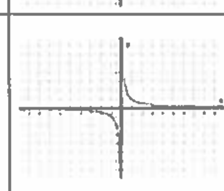
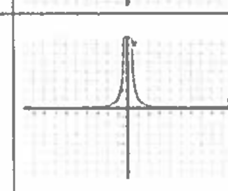
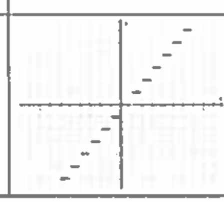
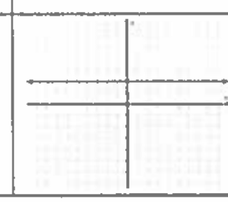
C The sound is equally loud at your position.

D The sound is four times as loud at your position.

E The sound is sixteen times as loud at your position.

Parent Graphs

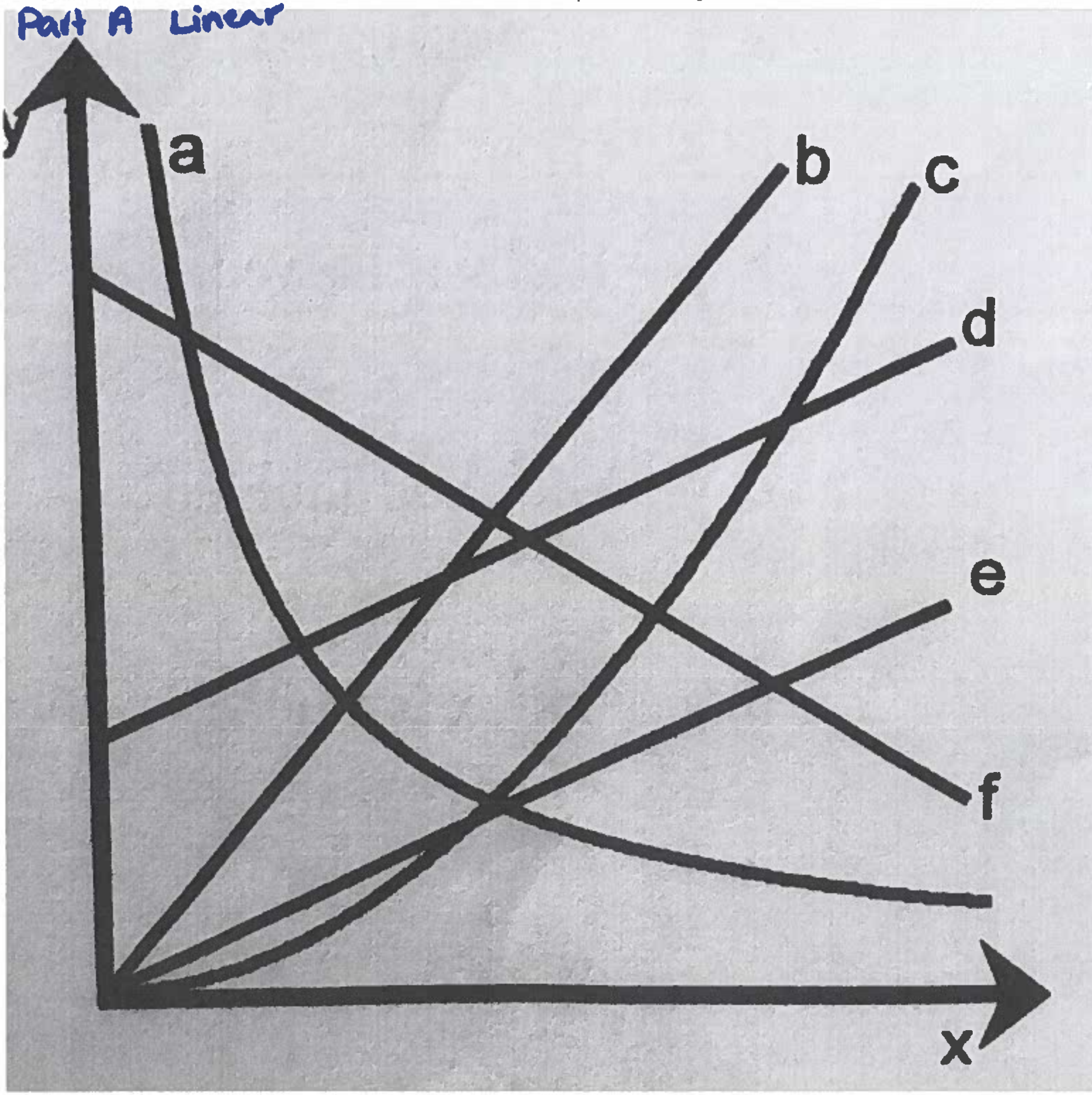
Learning Goal:
Parent graphs.

Parent Function	Graph	Parent Function	Graph
$y=x$ Linear Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$		$y=\lvert x\rvert$ Absolute Value Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$	
$y=x^2$ Quadratic Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$		$y=\sqrt{x}$ Square Root Neither Domain: $[0,\infty)$ Range: $[0,\infty)$	
$y=x^3$ Cubic Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$		$y=\sqrt[3]{x}$ Cube Root Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$	
$y=b^x,\ b>1$ Exponential Neither Domain: $(-\infty,\infty)$ Range: $(0,\infty)$		$y=\log_b(x),\ b>1$ Log Neither Domain: $(0,\infty)$ Range: $(-\infty,\infty)$	
$y=\frac{1}{x}$ Rational or Inverse Odd Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$		$y=\frac{1}{x^2}$ Inverse Squared Even Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(0,\infty)$	
$y=\text{int}(x)=[x]$ Greatest Integer Neither Domain: $(-\infty,\infty)$ Range: $\{y: y\in\mathbb{Z}\}$ (only integers)		$y=C$ Constant Function Even Domain: $(-\infty,\infty)$ Range: $\{y: y=C\}$	

Part A

The diagram shows a number of relationships between x and y.

Part A Linear



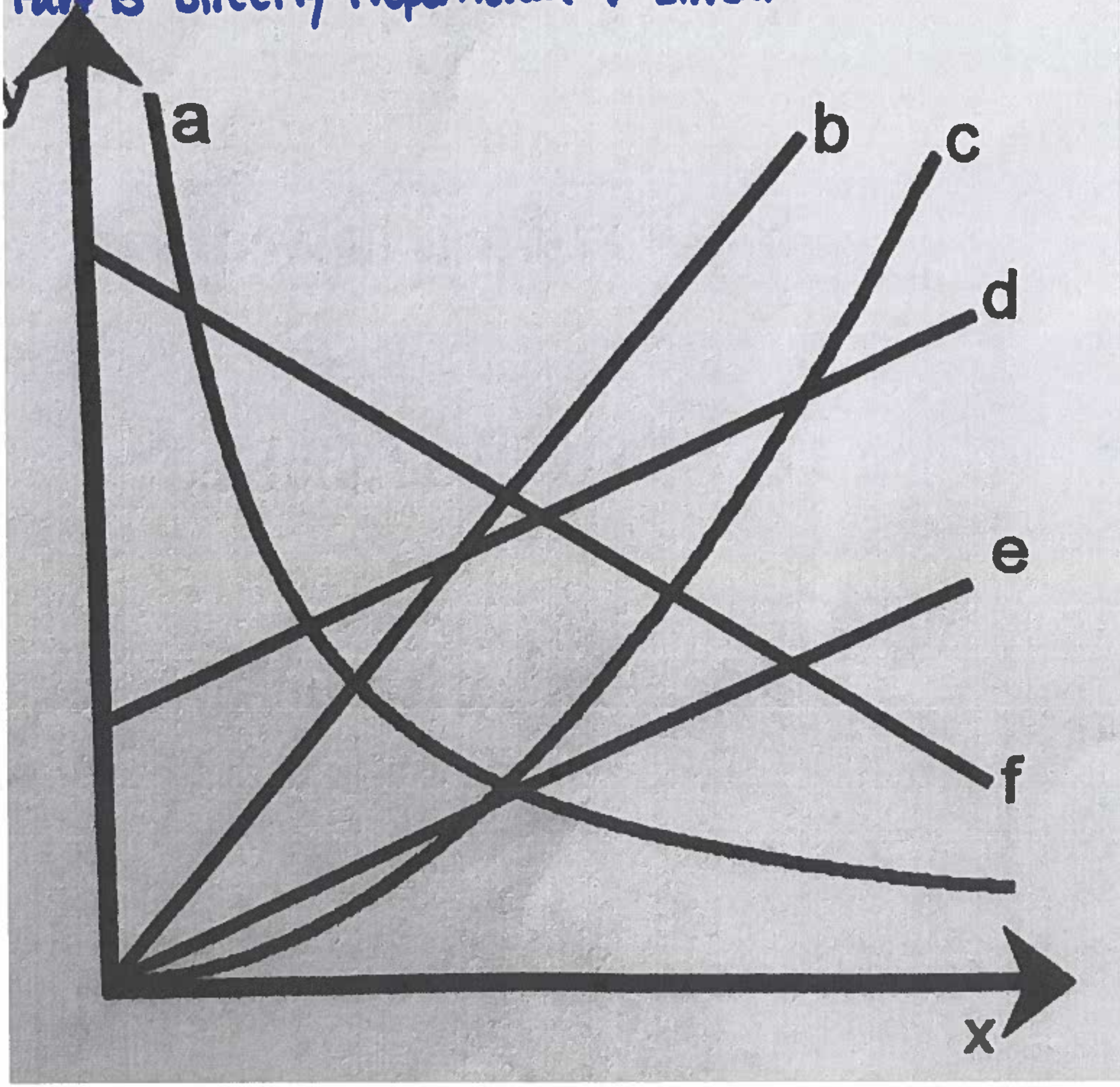
ANSWER:

7\. Which relationships are linear?

- a
- b
- c
- d
- e
- f

Part B - Directly Proportional and Linear Relationship

Part B Directly Proportional + Linear



The diagram shows a number of relationships between x and y.

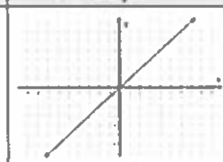
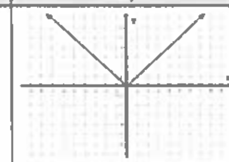
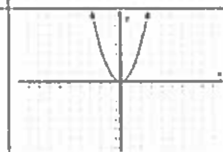
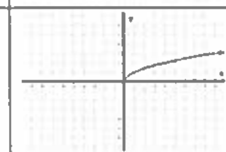
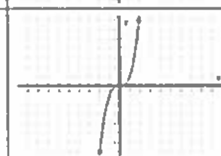
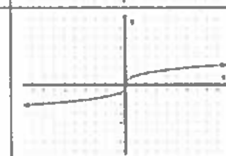
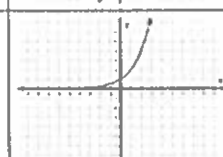
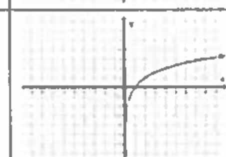
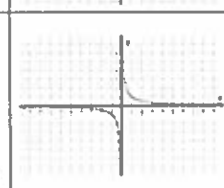
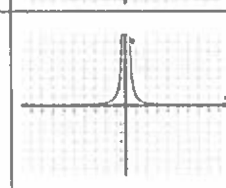
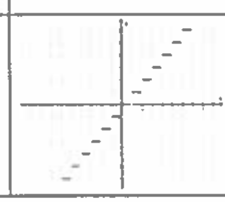
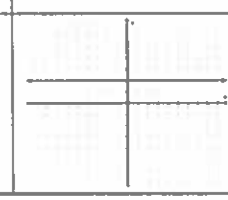
Being directly proportional and having a linear relationship are the same thing when the y-intercept is zero. That means b in y=mx+b is zero.

ANSWER:

8. Which relationships are direct proportions?

a
b
c
d
e
f

Review Parent Graphs

Parent Function	Graph	Parent Function	Graph
$y = x$ Linear Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$		$y = \lvert x \rvert$ Absolute Value Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$	
$y = x^2$ Quadratic Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$		$y = \sqrt{x}$ Square Root Neither Domain: $[0,\infty)$ Range: $[0,\infty)$	
$y = x^3$ Cubic Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$		$y = \sqrt[3]{x}$ Cube Root Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$	
$y = b^x,\ b > 1$ Exponential Neither Domain: $(-\infty,\infty)$ Range: $(0,\infty)$		$y = \log_b(x),\ b > 1$ Log Neither Domain: $(0,\infty)$ Range: $(-\infty,\infty)$	
$y = \frac{1}{x}$ Rational or Inverse Odd Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$		$y = \frac{1}{x^2}$ Inverse Squared Even Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(0,\infty)$	
$y = \text{int}(x) = [x]$ Greatest Integer Neither Domain: $(-\infty,\infty)$ Range: $\{y : y \in \mathbb{Z}\}$ (only integers)		$y = C$ Constant Function Even Domain: $(-\infty,\infty)$ Range: $\{y : y = C\}$	

Part C - Proportional Equations

Choose the following relationships that are direct proportions.

You did not open hints for this part.

ANSWER:

9.
A $y=1/x$
B $y=a/x$
C $y=ax$
D $y=ax^2$
E $y=x$
F $y=ax+b$
G $y=3x$
H $y=a/x^2$

Part D - Proportional Equations

Choose the following relationships that are linear relationships.

You did not open hints for this part.

ANSWER:

10.
A $y=ax^2$
B $y=ax$
C $y=3x$
D $y=ax+b$
E $y=1/x$
F $y=a/x$
G $y=a/x^2$
H $y=x$

Part E - Proportional Equations

Choose the following relationships that are quadratic relationships.

ANSWER:

11.
A $y=a/x$
B $y=a/x^2$
C $y=1/x$
D $y=ax^2$
E $y=ax$
F $y=3x$
G $y=x$
H $y=ax+b$

Part F - Proportional Equations

Choose the following relationships that are inverse square.

ANSWER:

12.

A y=a/x
B y=ax+b
C $y=a/x^2$
D y=1/x
E $y=ax^2$
F y=ax
G y=x
H y=3x

Part G - Mathematical Relationship

For each of the following mathematical relations, state what happens to the value of y when the following changes are made. (k is constant)

ANSWER:

13.

Reset Help

A **quadrupled (x4)**
B **quartered (1/4)**
C **x9**
D **tripled (x3)**
E **halved (1/2)**
F **doubled (x2)**

1. y = kx, x is tripled. y is ______ .
2. $y = k/x^2$, x is doubled. y is ______ .
3. $y = kx^2$, x is tripled. y is ______ .
4. y = k/x, x is halved. y is ______ .